JEE Advanced

Single Correct Answer Type

1. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of the diameter of the ring. The wheel now rotates with an angular velocity

   a. $\frac{\omega M}{(M+m)}$
   b. $\frac{\omega(M-2m)}{(M+2m)}$
   c. $\frac{\omega M}{(M+2m)}$
   d. $\frac{\omega(M+2m)}{M}$ **(IIT-JEE 1983)**

2. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum is located at a distance of

   a. 0.42 m from the mass of 0.3 kg
   b. 0.70 m from the mass of 0.7 kg
   c. 0.98 m from the mass of 0.3 kg
   d. 0.98 m from the mass of 0.7 kg **(IIT-JEE 1995)**

3. A mass m is moving with a constant velocity along a line parallel to the x-axis, away from the origin. Its angular momentum with respect to the origin

a. is zero
b. remains constant
c. goes on increasing
d. goes on decreasing

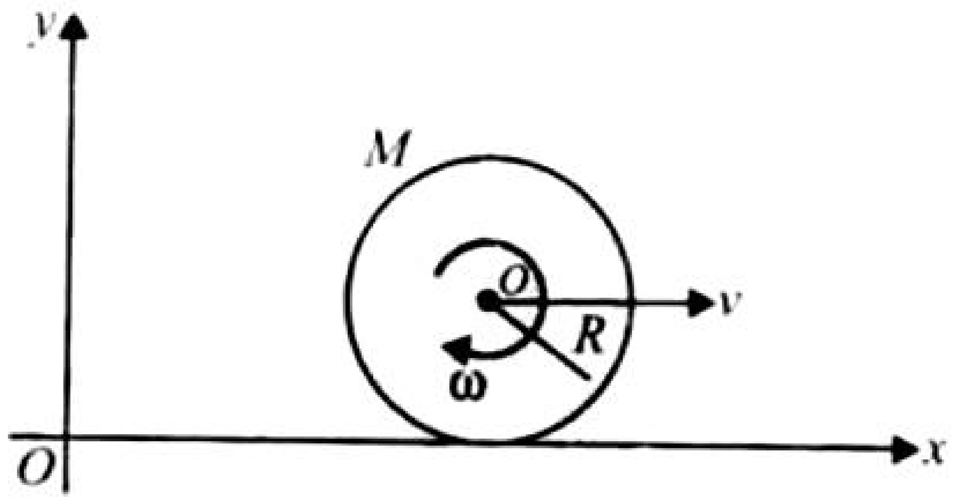
(IIT-JEE 1997)

4. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω_A and ω_B, respectively. Then

a. $\omega_A < \omega_B$
b. $\omega_A = \omega_B$
c. $\omega_A = \omega$
d. $\omega_B = \omega$

(IIT-JEE 1999)

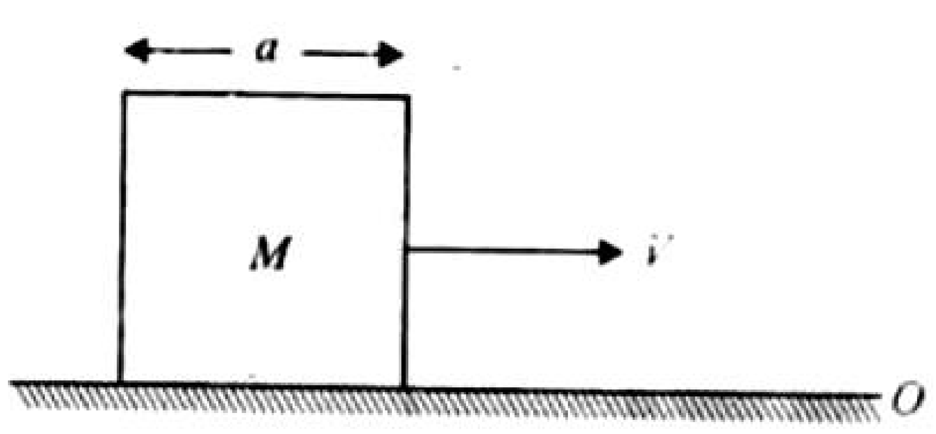
5. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is



a. $(1/2)\,MR^2\omega$
b. $MR^2\omega$
c. $(3/2)\,MR^2\omega$
d. $2\,Mr^2\omega$

(IIT-JEE 1999)

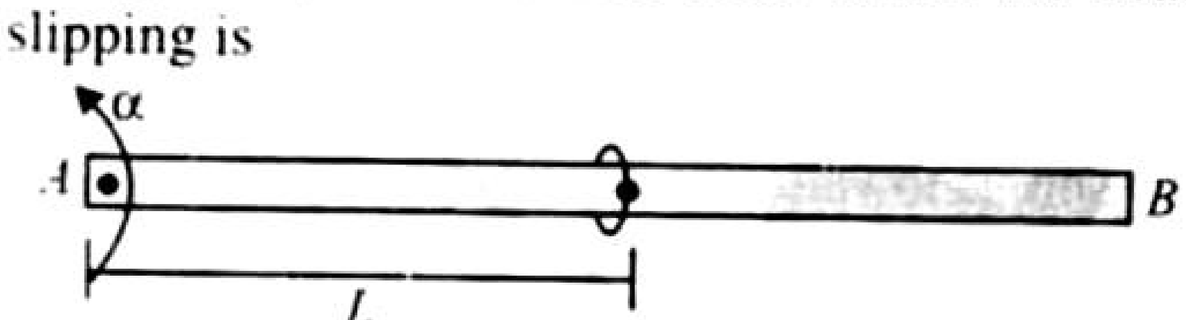
6. A cubical block of side a is moving with velocity V on a horizontal smooth plane as shown in figure. It hits a ridge at point O. The angular speed of the block after it hits O is



a. $\frac{3V}{4a}$
b. $\frac{3V}{2a}$
c. $\frac{\sqrt{3}V}{\sqrt{2}a}$
d. zero

(IIT-JEE 1999)

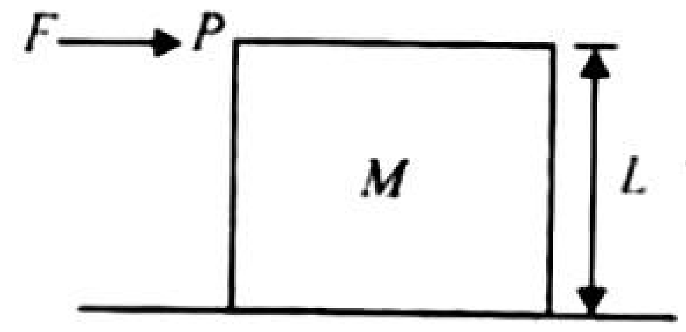
7. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is



a. $\sqrt{\frac{\mu}{\alpha}}$
b. $\frac{\mu}{\sqrt{\alpha}}$
c. $\frac{1}{\sqrt{\mu\alpha}}$
d. infinitesimal

(IIT-JEE 2000)

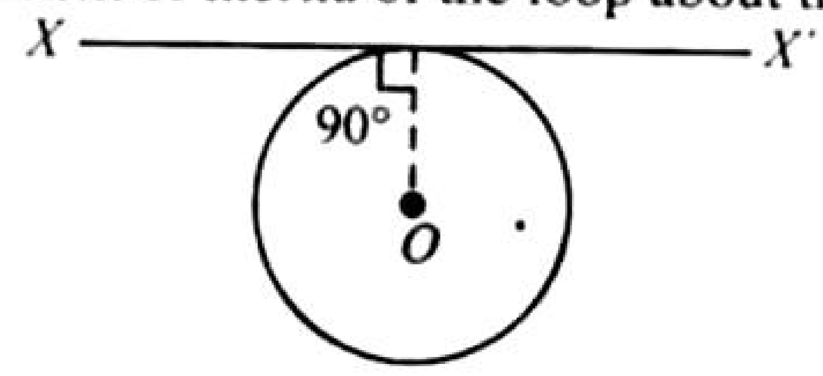
8. A cubical block of side L rests on a rough horizontal surface with coefficient of friction sufficiently high so that the block does not slide before toppling; the minimum force required to topple the block is



a. infinitesimal
b. mg 4
c. $mg/2$
d. $mg(1-\mu)$

(IIT-JEE 2000)

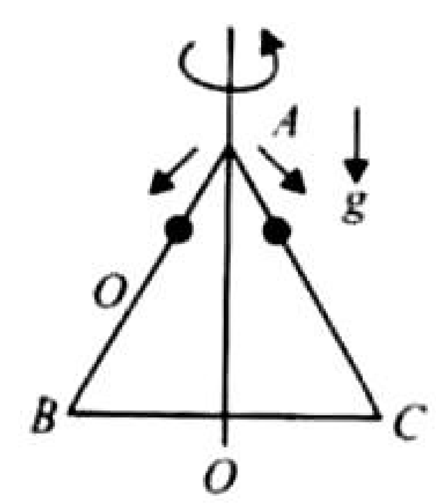
9. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



a. $\frac{\pi L^3}{8\pi^2}$
b. $\frac{\pi L^3}{16\pi^2}$
c. $\frac{5\pi L^3}{16\pi^2}$
d. $\frac{3L^3\rho}{8\pi^2}$

(IIT-JEE 2000)

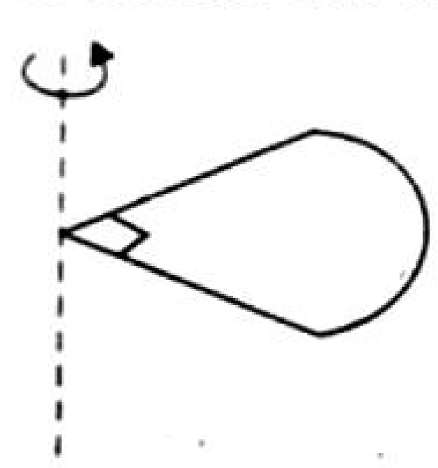
10. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down are



a. angular velocity and total energy (kinetic and potential)
b. total angular momentum and total energy
c. angular velocity and moment of inertia about the axis of rotation
d. total angular momentum and moment of inertia about the axis of rotation

(IIT-JEE 2000)

11. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the centre of the original disc. Its moment of inertia about the axis of rotation is



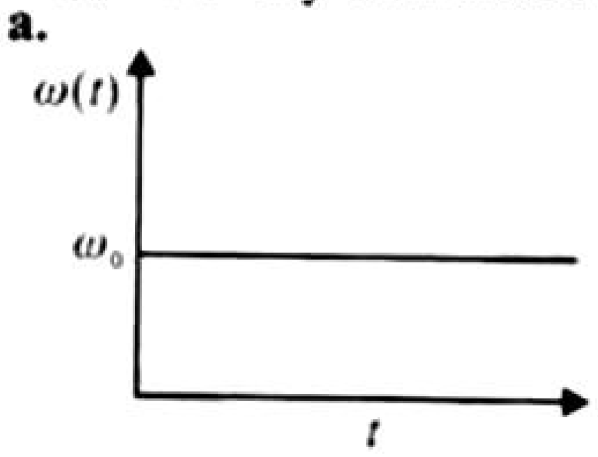
a. $\frac{1}{2}MR^2$
b. $\frac{1}{4}MR^2$
c. $\frac{1}{8}MR^2$
d. $\sqrt{2}\,MR^2$

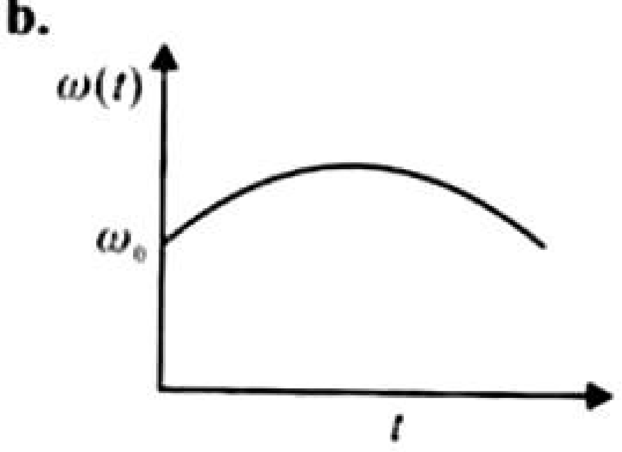
(IIT-JEE 2001)

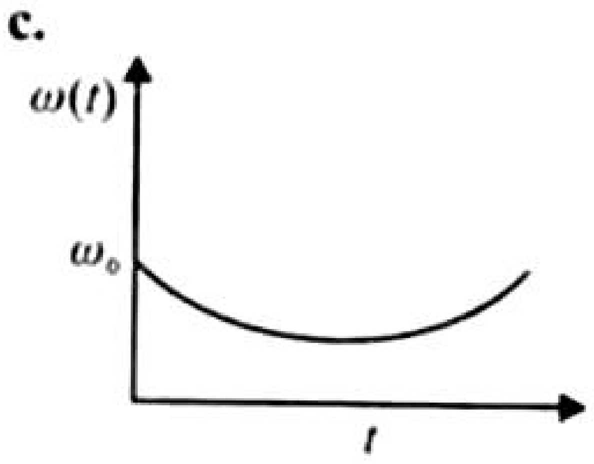
12. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are
   a. up the incline while ascending and down the incline while descending
   b. up the incline while ascending as well as descending
   c. down the incline while ascending and up the incline while descending
   d. down the incline while ascending as well as descending

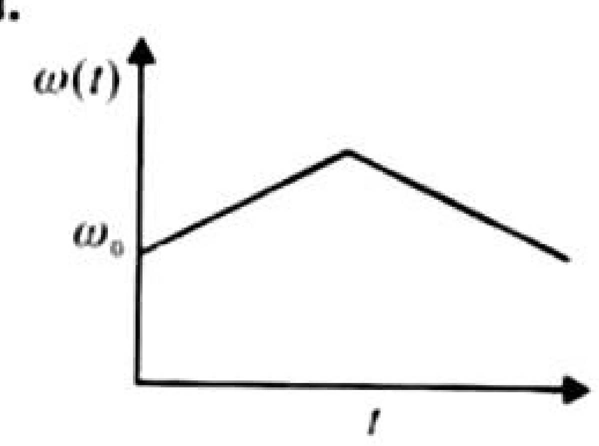
(IIT-JEE 2002)

13. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

a.


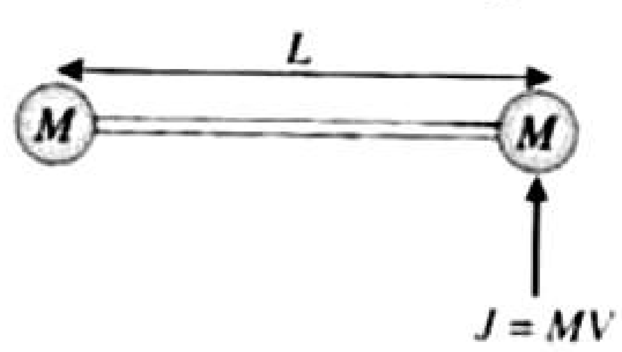
b.


c.


d.


(IIT-JEE 2002)

14. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = MV$ is imparted to the body at one of its ends, what would be its angular velocity?



   a. $\frac{V}{L}$
   b. $\frac{2V}{L}$
   c. $\frac{V}{3L}$
   d. $\frac{V}{4L}$

(IIT-JEE 2003)

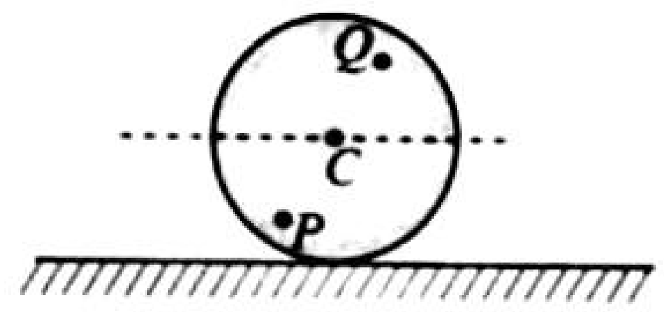
15. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?
   a. Centre of the circle
   b. On the circumference of the circle
   c. Inside the circle
   d. Outside the circle

(IIT-JEE 2003)

16. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_o. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be
   a. $2K$
   b. $K/2$
   c. K
   d. $K/4$

(IIT-JEE 2004)

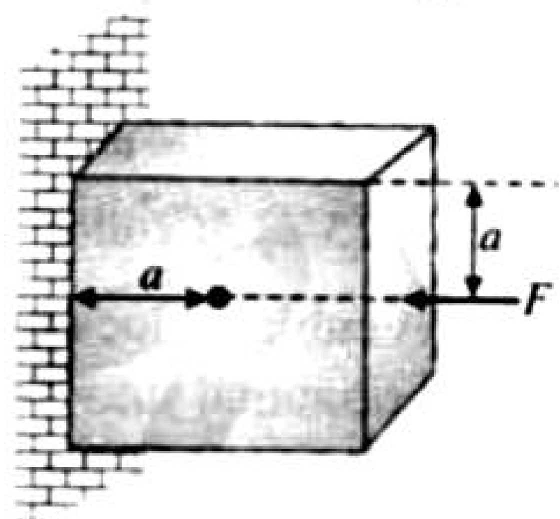
17. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is



   a. $V_Q > V_C > V_P$
   b. $V_P > V_C > V_Q$
   c. $V_Q = V_P$, $V_C = V_P/2$
   d. $V_P < V_C > V_Q$

(IIT-JEE 2004)

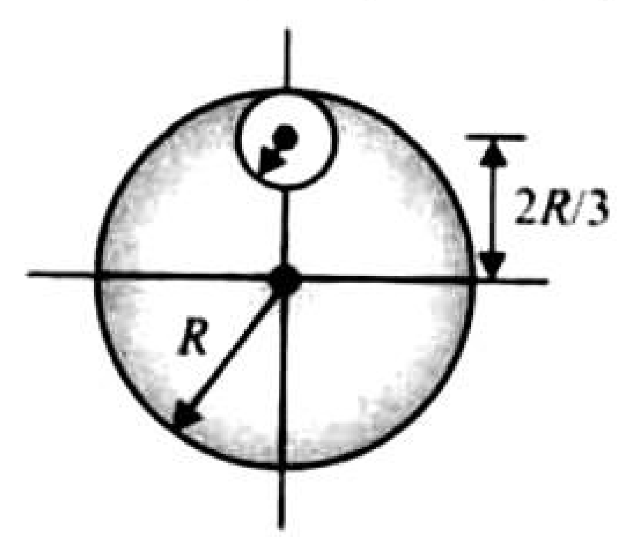
18. A block of mass m is at rest under the action of F against the wall as shown in figure. Which of the following statement is incorrect?



   a. $f = mg$ [where f is the friction force]
   b. $F = N$ [where N is the normal force]
   c. F will not produce torque
   d. N will not produce torque

(IIT-JEE 2005)

19. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



   a. $4MR^2$
   b. $\frac{40}{4} MR^2$
   c. $10MR^2$
   d. $\frac{37}{9} MR^2$

(IIT-JEE 2005)

20. A particle is confined to rotate in a circular path decreasing linear speed, which of the following is correct?
   a. $\vec{L}$ (angular momentum) is conserved about the centre
   b. Only direction of angular momentum $\vec{L}$ is conserved
   c. It spirals towards the centre
   d. Its acceleration is towards the centre

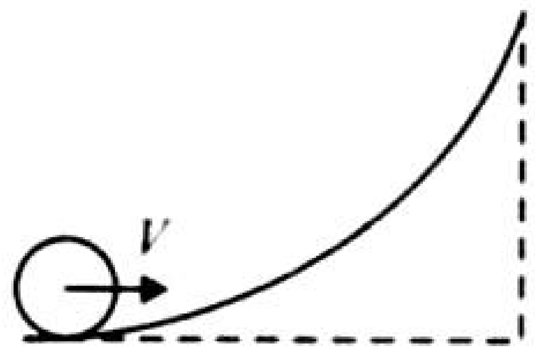
(IIT-JEE 2005)

21. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as

a. $r=\sqrt{\frac{2}{15}}R$ b. $r=\sqrt{\frac{2}{15}}R$

c. $r=\frac{2}{15}R$ d. $r=\frac{2}{\sqrt{5}}R$ **(IIT-JEE 2006)**

22. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2/4g$ with respect to the initial position. The object is a



a. ring b. solid sphere

c. hollow sphere d. disc

(IIT-JEE 2007)

23. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

a. linear momentum of the system does not change in time

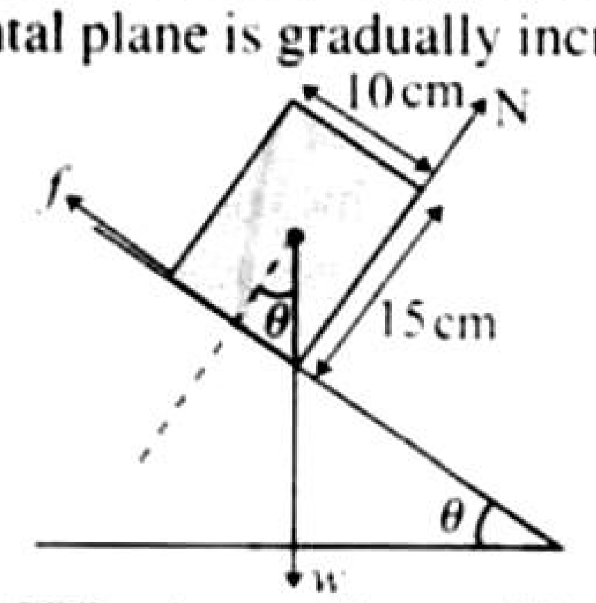
b. kinetic energy of the system does not change in time

c. angular momentum of the system does not change in time

d. potential energy of the system does not change in time

(IIT-JEE 2009)

24. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is 3. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then



FBD at just toppling condition

a. at $\theta = 30°$, the block will start sliding down the plane

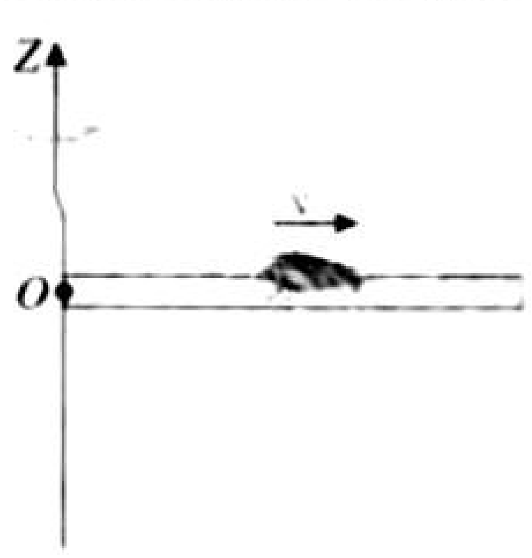
b. the block will remain at rest on the plane up to certain θ and then it will topple

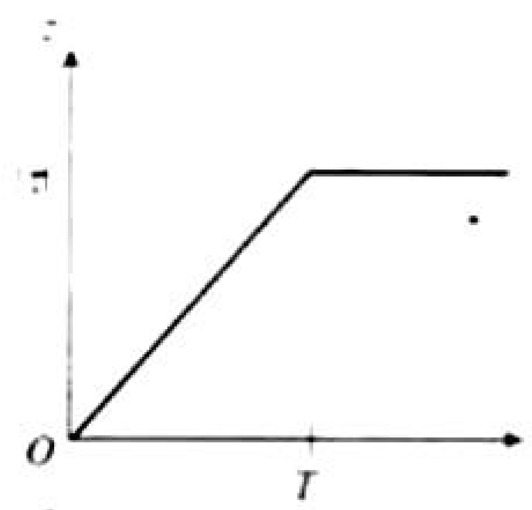
c. at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

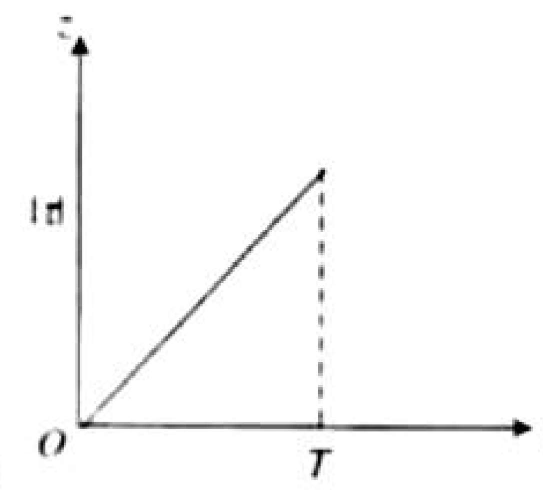
d. at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

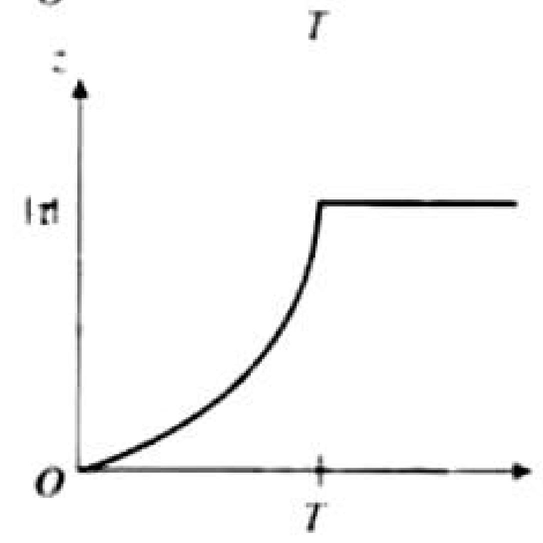
(IIT-JEE 2009)

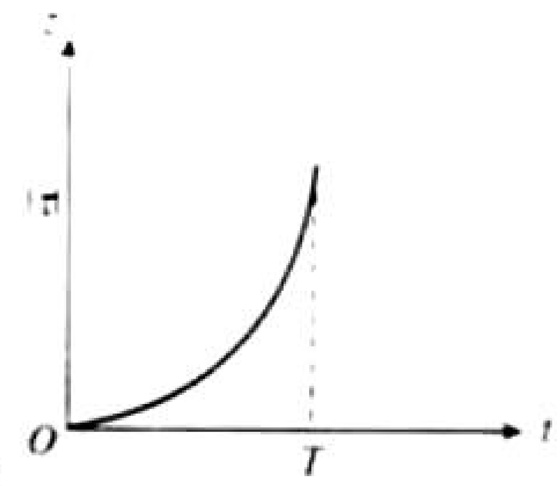
25. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($\vec{\tau}$) on the system about O, as a function of time is best represented by which plot?



a. 

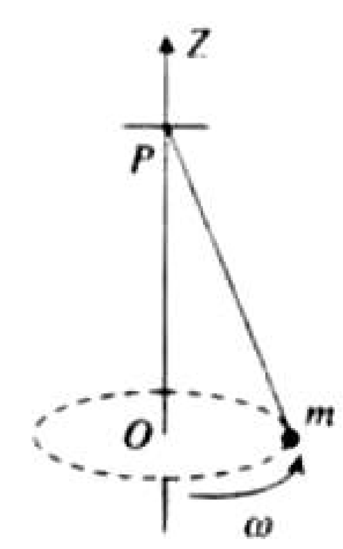
b. 

c. 

d. 

(IIT-JEE 2012)

26. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x–y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



a. $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.

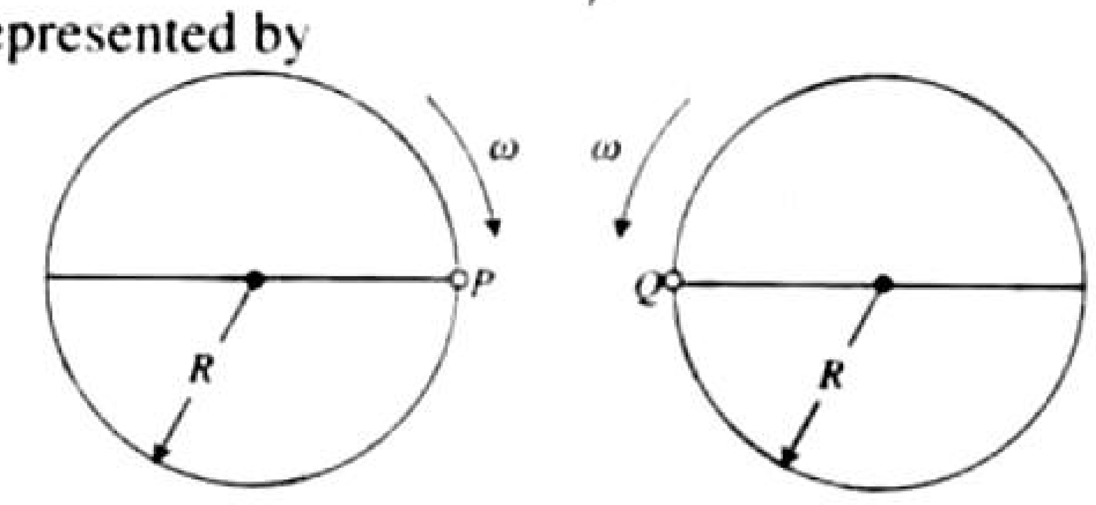
b. $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

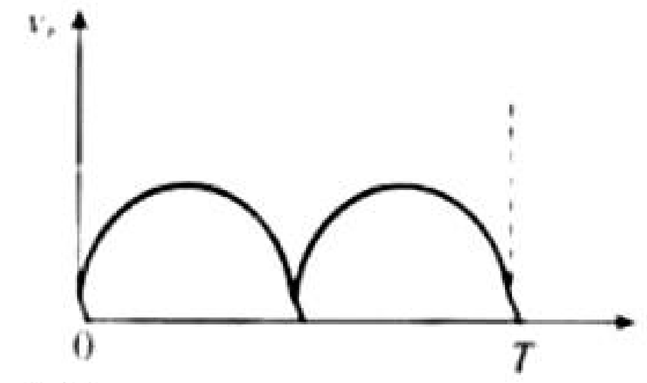
c. $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time.

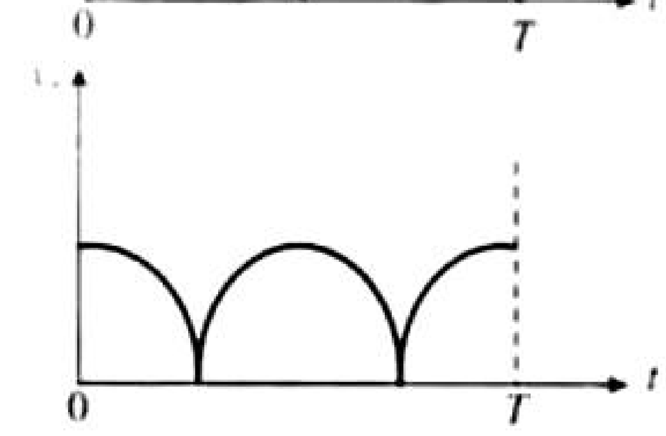
d. $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

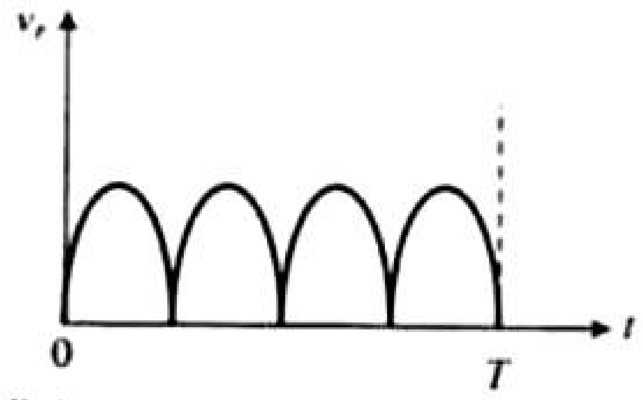
(IIT-JEE 2012)

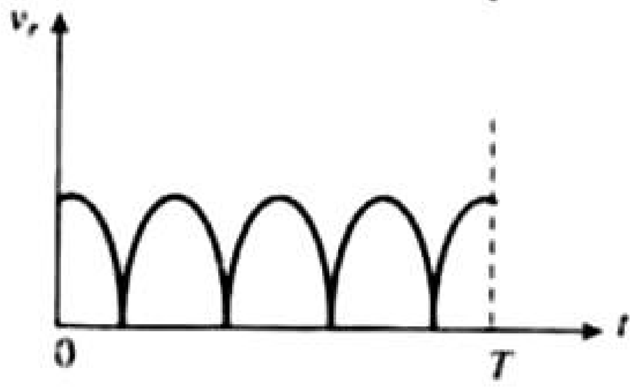
27. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



a. 

b. 

c. 

d. 

(IIT-JEE 2012)

28. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is/are correct?

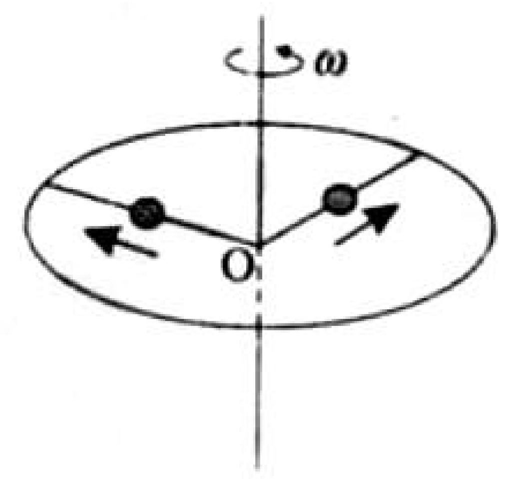
a. Both cylinders P and Q reach the ground at the same time

b. Cylinder P has larger linear acceleration than cylinder Q

c. Both cylinders reach the ground with same translational kinetic energy.

d. Cylinder Q reaches the ground with larger angular speed. **(IIT-JEE 2012)**

29. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is:



a. $\frac{2}{3}R$ **b.** $\frac{1}{3}R$ c. $\frac{3}{5}R$ **d.** $\frac{4}{5}R$

(JEE Advanced 2015)

Multiple Correct Answers Type

1. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is V and the speed of B is $2V$, the speed of the centre of mass of the system is

a. $3V$ **b.** V c. $1.5V$ **d.** zero

(IIT-JEE 1982)

2. A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin

a. is zero b. remains constant

c. goes on increasing d. goes on decreasing

(IIT-JEE 1995)

3. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts

a. in the backward direction on the front wheel and in the forward direction on the rear wheel

b. in the forward direction on the front wheel and in the backward direction on the rear wheel

c. in the backward direction on both the front and the rear wheels

d. in the forward direction on both the front and the rear wheels **(IIT-JEE 1990)**

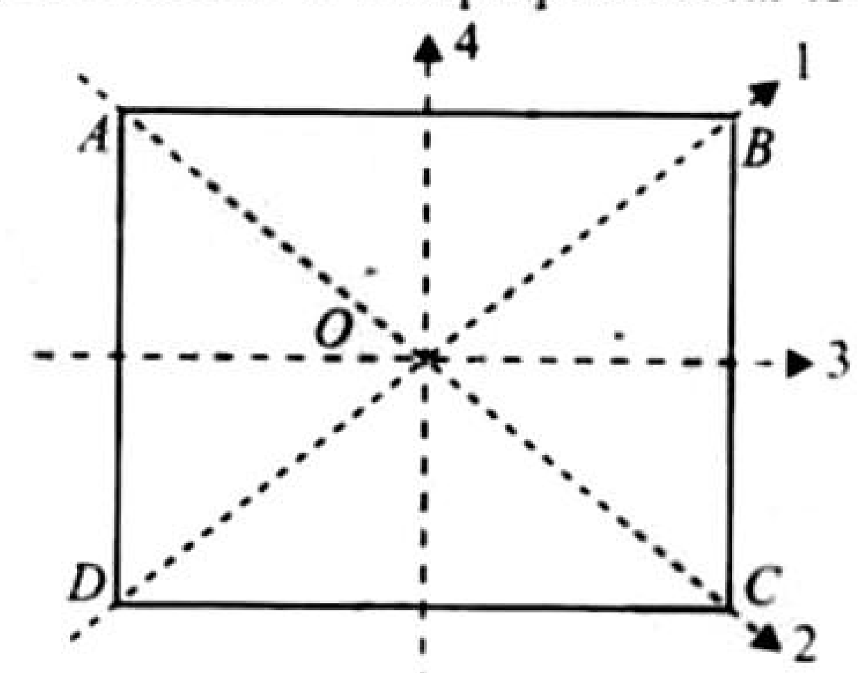
4. A particle of mass m is projected with a velocity V making an angle of 45° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle at its maximum height h is

a. zero **b.** $\frac{mv^3}{4\sqrt{2}g}$

c. $\frac{mV^3}{\sqrt{2}g}$ **d.** $\frac{mV}{\sqrt{2gh^3}}$

(IIT-JEE 1990)

5. The moment of inertia of a thin square plate $ABCD$, see figure, of uniform thickness about an axis passing through the centre O and perpendicular to the plane is



a. $I_1 + I_2$ **b.** $I_3 + I_4$

c. $I_1 + I_3$ **c.** $I_1 + I_2 + I_3 + I_4$

(IIT-JEE 1992)

6. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

a. $\frac{M\omega^2 L}{2}$ **b.** $M\omega^2 L$

c. $\frac{M\omega^2 L}{4}$ **d.** $\frac{M\omega^2 L^2}{4}$

(IIT-JEE 1992)

7. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with the track is

a. zero b. 30° c. 45° d. 60°

(IIT-JEE 1992)

8. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

a. I b. $I\sin^2\theta$

c. $I\cos^2\theta$ d. $I\cos^2(\theta/2)$

(IIT-JEE 1998)

9. The torque τ on a body about a given point is found to be equal to $\mathbf{A}\times\mathbf{L}$ where $\mathbf{A}$ is a constant vector, and $\mathbf{L}$ is the angular momentum of the body about that point. From this it follows that

a. $d\mathbf{L}/dt$ is perpendicular to $\mathbf{L}$ at all instants of time

b. the component of $\mathbf{L}$ in the direction of A does not change with time

c. the magnitude of $\mathbf{L}$ does not change with time

d. $\mathbf{L}$ does not change with time **(IIT-JEE 1998)**

10. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then

a. the friction force is dissipative

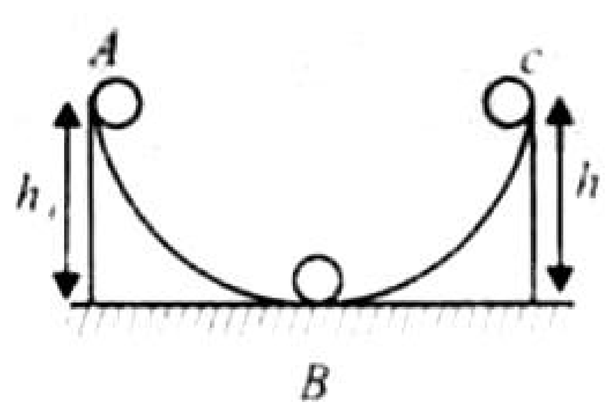
b. the friction force is necessarily changing

c. the friction force will aid rotation but hinder translation

d. the friction force is reduced if θ is reduced

(IIT-JEE 2006)

11. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then

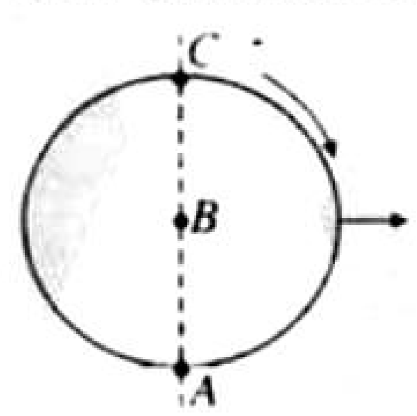


a. $h_A > h_C$; $K_B > K_C$ b. $h_A > h_C$; $K_C > K_A$

c. $h_A = h_C$; $K_B = K_C$ d. $h_A < h_C$; $K_B > K_C$

(IIT-JEE 2006)

12. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

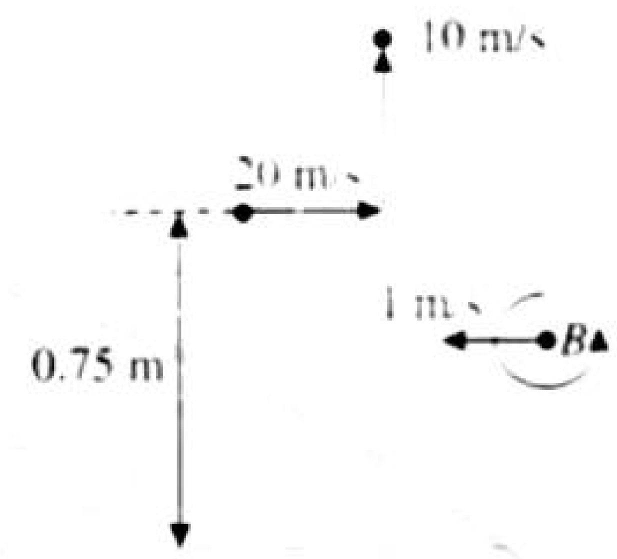


a. $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ b. $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

c. $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ d. $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

(IIT-JEE 2009)

13. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



a. the ring has pure rotation about its stationary

b. the ring comes to a complete stop

c. friction between the ring and the ground is to the left

d. there is no friction between the ring and the ground

(IIT-JEE 2011)

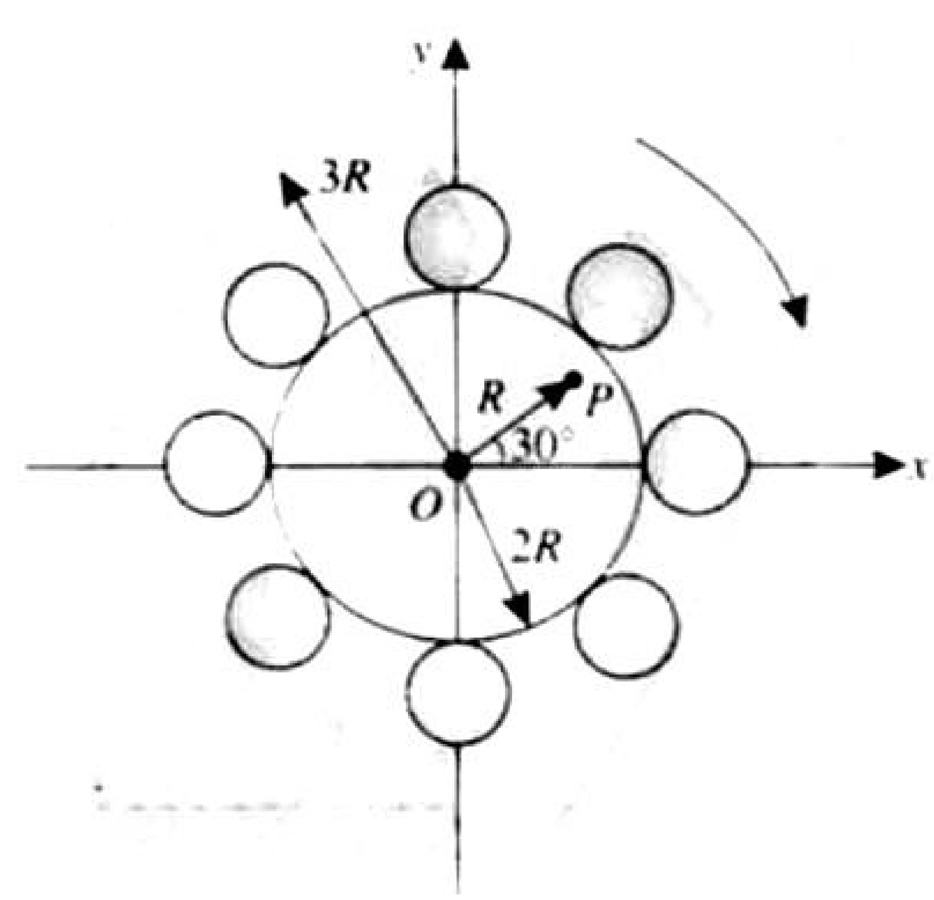
14. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then

a. $V_Q > V_R > V_P$ b. $V_R > V_Q > V_P$

c. $V_R/V_P = 3$ d. $V_P/V_Q = 1/2$

(IIT-JEE 2012)

15. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x–z plane. The point P on the inner disc is at distance R from the origin, where OP makes an angled of 30° with the horizontal. Then with respect to the horizontal surface,



a. The point O has a linear velocity $3R\omega\hat{i}$

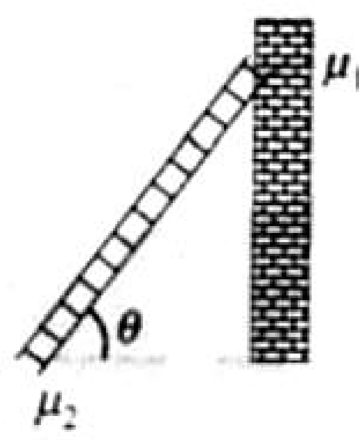
b. The point P has a linear velocity

$\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

c. The point P has a linear velocity $\frac{13}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

d. The point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$ **(IIT-JEE 2012)**

16. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



a. $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = mg/2$
b. $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = mg/2$
c. $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$
d. $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = mg/2$

(JEE Advanced 2014)

Linked Comprehension Type

For Problems 1–3

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$, respectively, about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

(IIT-JEE 2007)

1. The ratio x_1/x_2 is

a. 2 b. $\frac{1}{2}$
c. $\sqrt{2}$ d. $\frac{1}{\sqrt{2}}$

2. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

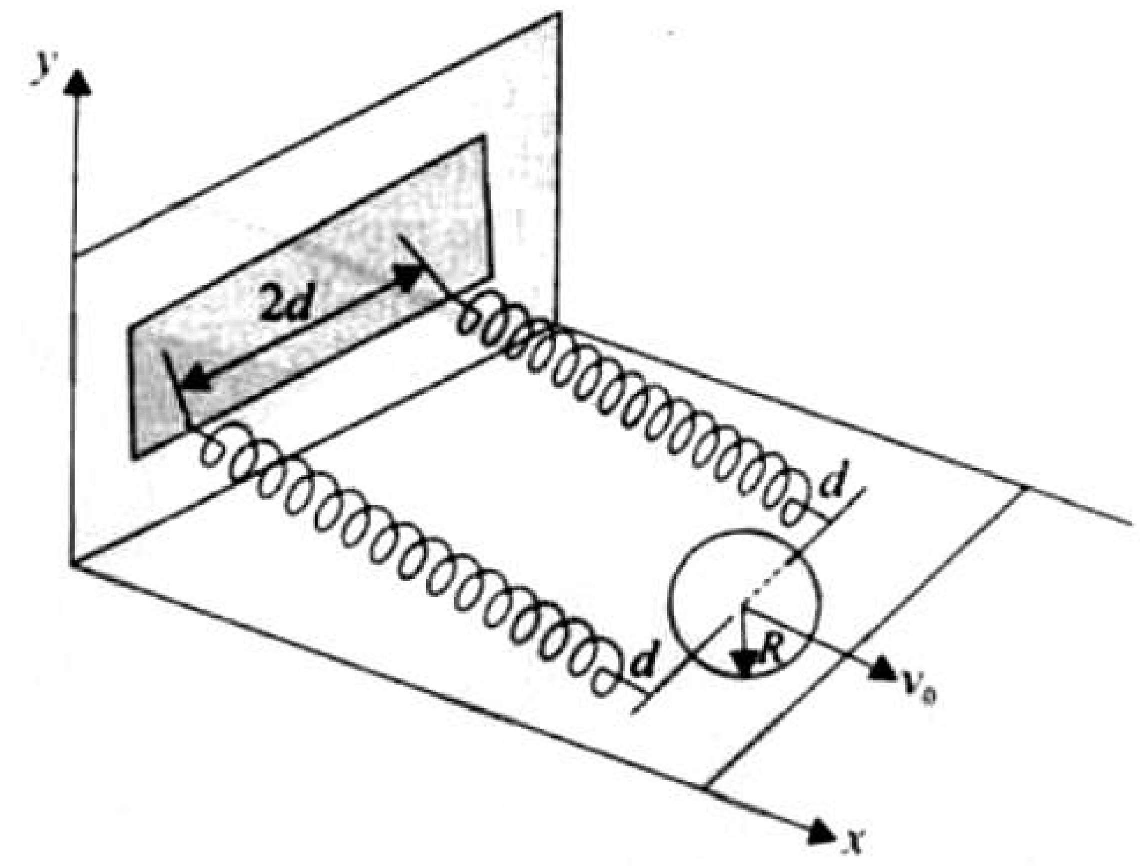
a. $\frac{2I\omega}{3t}$ b. $\frac{9I\omega}{2t}$
c. $\frac{9I\omega}{4t}$ d. $\frac{3I\omega}{2t}$

3. The loss of kinetic energy during the above process is

a. $\frac{I\omega^3}{2}$ b. $\frac{I\omega^2}{3}$
c. $\frac{I\omega^2}{4}$ d. $\frac{I\omega^2}{6}$

For Problems 4–6

A uniform thin cylindrical disc of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in horizontal plane. The unstretched length of each spring is L. The disc is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disc rolls without slipping with velocity $V_0 = V_0\hat{i}$. The coefficient of friction is μ.



(IIT-JEE 2008)

4. The net external force acting on the disc when its centre of mass is at displacement x with respect to its equilibrium position is

a. $-kx$ b. $-2kx$
c. $-\frac{2kx}{3}$ d. $-\frac{4kx}{3}$

5. The centre of mass of the disc undergoes simple harmonic motion with angular frequency ω equal to

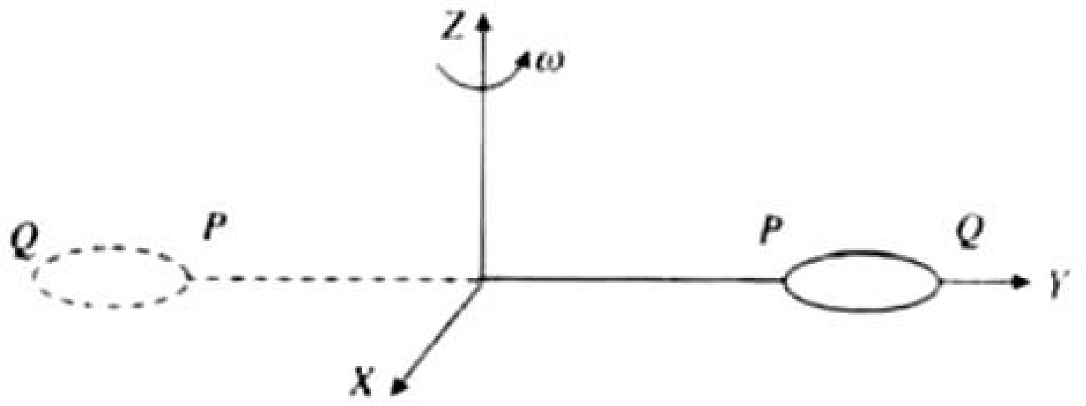
a. $\sqrt{\frac{k}{M}}$ b. $\sqrt{\frac{2k}{M}}$ c. $\sqrt{\frac{2k}{3M}}$ d. $\sqrt{\frac{4k}{3M}}$

6. The maximum value of V_0 for which the disc will roll without slipping is

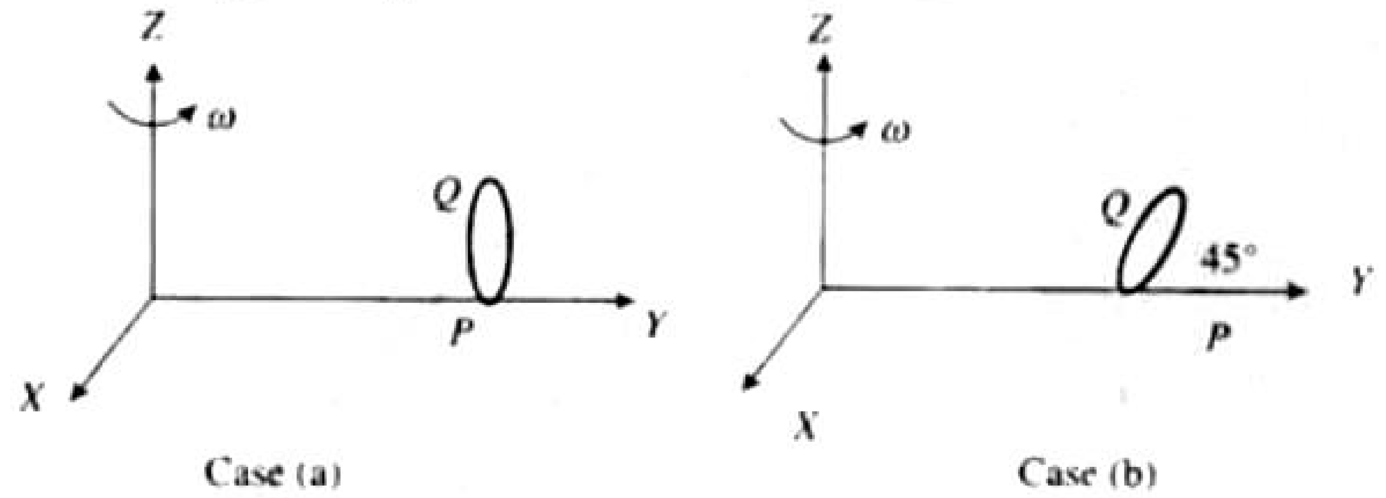
a. $\mu g\sqrt{\frac{M}{k}}$ b. $\mu g\sqrt{\frac{M}{2k}}$
c. $\mu g\sqrt{\frac{3M}{k}}$ d. $\mu g\sqrt{\frac{5M}{2k}}$

For Problems 7–8

The general motion of a rigid body can be considered to be a combination of (i) a motion.... of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc–stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure: case (a) the disc with its face vertical and parallel to x–z plane; case (b) the disc with its face making an angle of 45° with x–y plane and its horizontal diameter parallel to x–axis. In both the cases, the disc is welded at point P, and the system are rotated with constant angular speed ω about the z-axis.



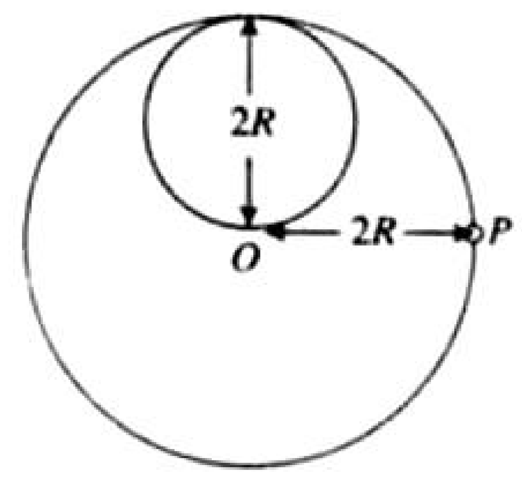
(IIT-JEE 2012)

7. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?
   a. It is $\sqrt{2}\,\omega$ for both the cases
   b. It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)
   c. It is ω force case (a); and $\sqrt{2}\,\omega$ for case (b)
   d. It is ω for both the cases
8. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?
   a. It is vertical for both the cases (a) and (b)
   b. It is vertical for case (a); and is at 45° to the x–z plane and lies in the plane of the disc for case (b)
   c. It is horizontal for case (a); and is at 45° to the x–z plane and is normal to the plane of the disc for case (b)
   d. It is vertical for case (a); and is at 45° to the x–z plane and is normal to the plane of the disc for case (b)

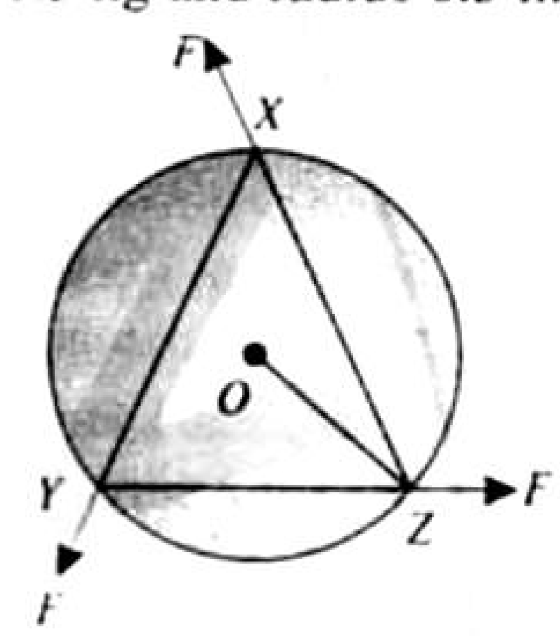
Integer Answer Type

1. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[IIT-JEE 2011]**

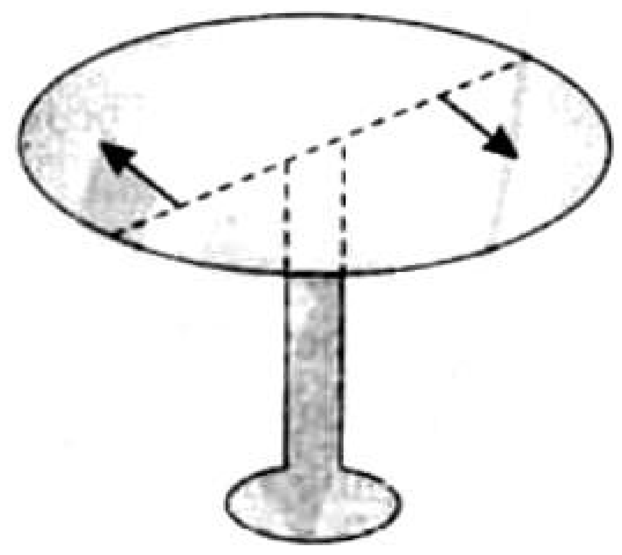
2. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is **[IIT-JEE 2011]**
3. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_P/I_O to the nearest integer is. **(IIT-JEE 2012)**



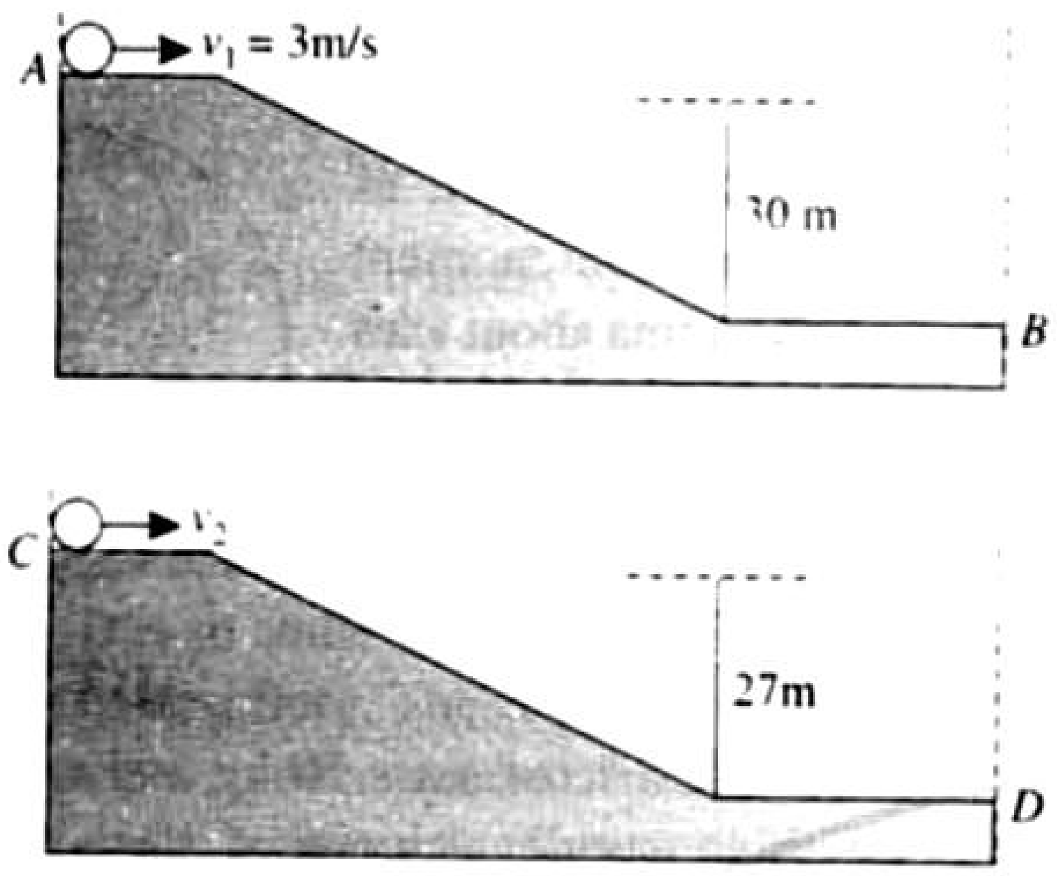
4. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad/s about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is. **(JEE Advanced 2013)**
5. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **(JEE Advanced 2014)**



6. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **(JEE Advanced 2014)**



7. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is ______. (g = 10 m/s^2) **(JEE Advanced 2015)**



8. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is: **(JEE Advanced 2015)**

Fill in the Blanks Type

a. Statement 1 is true, Statement 2 is true; Statement 2 is the correct explanation for Statement 1.

b. Statement 1 is true, Statement 2 is true; Statement 2 is NOT the correct explanation for Statement 1.

c. Statement 1 is true but Statement 2 is false.

d. Statement 1 is false but Statement 2 is true.

1. Statement 1: If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Statement 2: The linear momentum of an isolated system remains constant. **(IIT-JEE 2007)**

2. Statement 1: Two cylinders, one hollow (metal) and the other solid (wood), with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

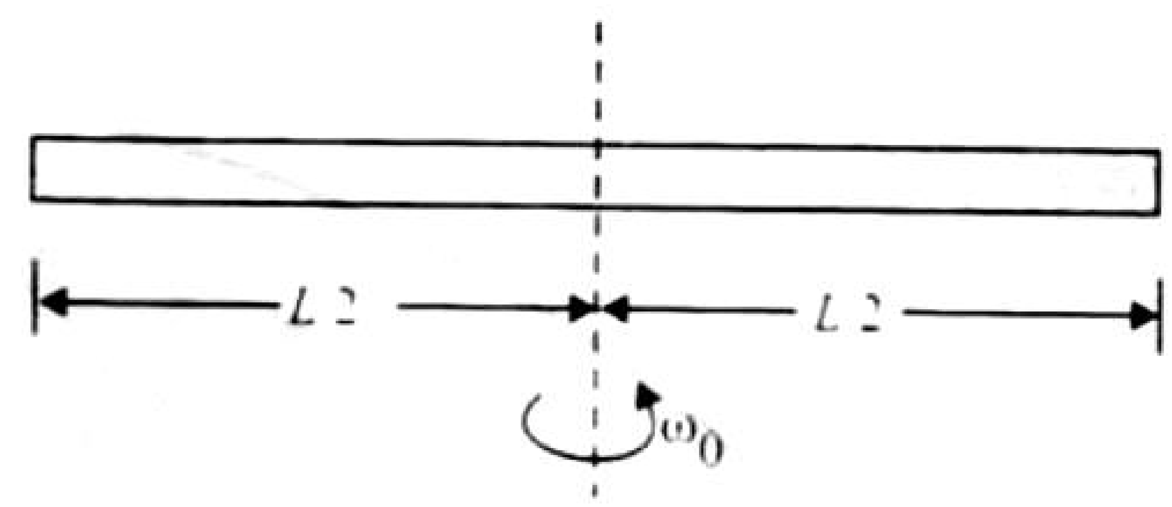
Statement 2: By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(IIT-JEE 2008)

Fill in the Blanks Type

1. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $3a/4$ above the base. The minimum value of F for which the cube begins to tip about the edge is____(Assume that the cube does not slide) **(IIT-JEE 1984)**

2. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the midpoint of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is_____.



(IIT-JEE 1988)

3. A cylinder of mass M and radius R is resting on a horizontal platform (which is parallel to the x-y plane) with its axis fixed along the y-axis and free to rotate about its axis. The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and the platform. The maximum torque acting on the cylinder during its motion is_____.

(IIT-JEE 1988)

4. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle (neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle and n = ___.

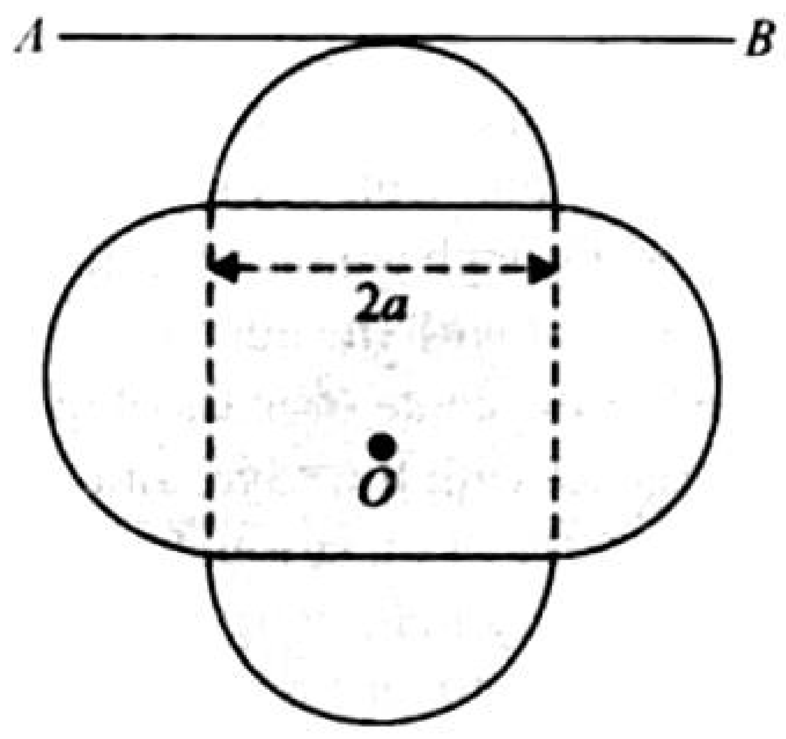
(IIT-JEE 1993)

5. A uniform disc of mass m and radius R is rolling up a rough inclined plane which makes an angle of 30° with the horizontal. If the coefficients of static and kinetic friction are each equal to μ and the only force acting are gravitational and frictional, then the magnitude of the frictional force acting on the disc is___and its direction is_____ (write up or down) the inclined plane.

(IIT-JEE 1997)

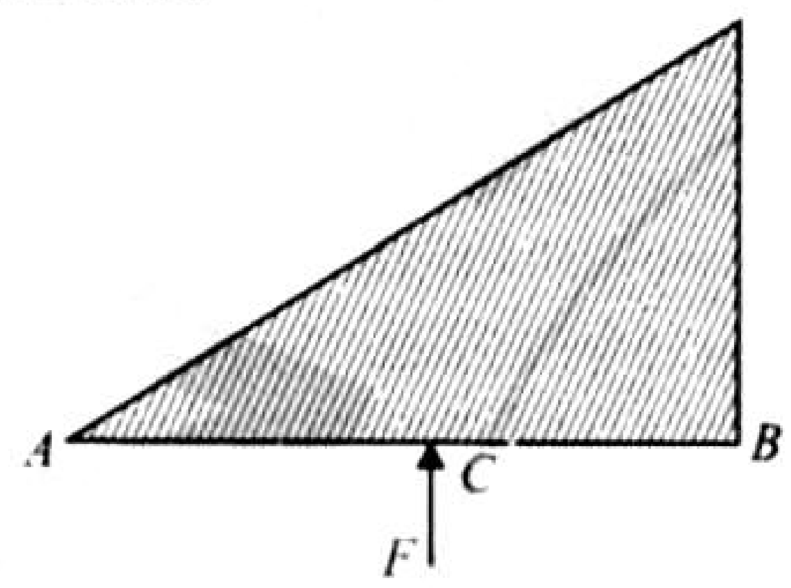
6. A rod of weight w is supported by two parallel knife-edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is _____ and on B is_____. **(IIT-JEE 1999)**

7. A symmetric lamina of mass M consists of a square shape with a semicircular section over of the edge of the square as shown in the figure. The side of the square is $2a$. The moment of inertia of the lamina about an axis through its centre of mass and perpendicular to the plane is $1.6Ma^2$. The moment of inertia of the lamina about the tangent AB in the plane of the lamina is_____. **(IIT-JEE 1997)**



True/False Type

1. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper and (a) passing through A, (b) passing through B, by the application of the same force, F, at C (midpoint of AB) as shown in the figure. The angular acceleration in both the cases will be the same.

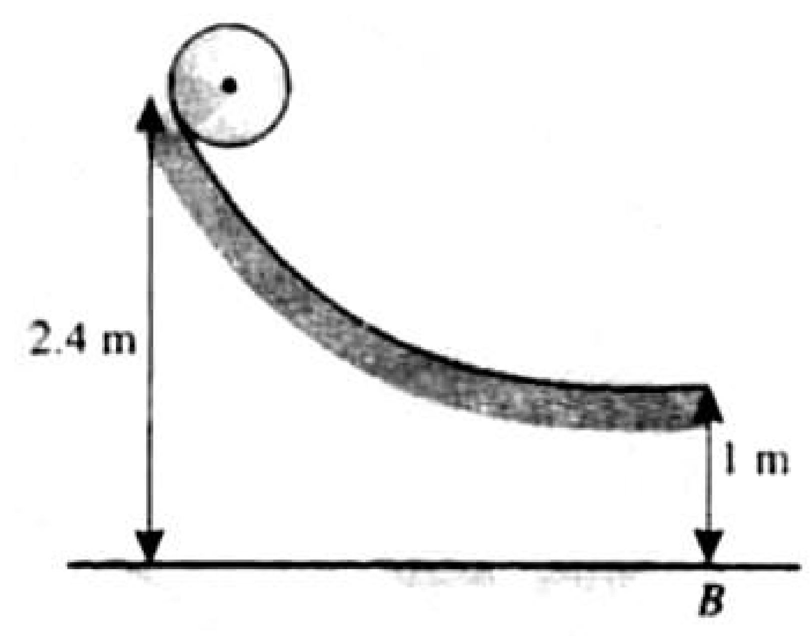


(IIT-JEE 1985)

2. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of the same dimensions but of mass $M/4$ is placed gently on the first disc coaxially. The angular velocity of the system now is $2\omega/\sqrt{5}$. **(IIT-JEE 1986)**
3. A ring of masses 0.3 kg and radius 0.1 m and a solid cylinder of mass 0.4 kg and of the same radius have the same kinetic energy and are released simultaneously on a flat horizontal surface such that they begin to roll as soon as they are released towards a wall which is at the same distance from the ring and the cylinder. The rolling friction in both the cases is negligible. The cylinder will reach the wall first. **(IIT-JEE 1989)**
4. Two particles of mass 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centre of mass has a velocity of 0.5 m/s, the velocity of the centre of mass is 0.75 m/s. **(IIT-JEE 1989)**

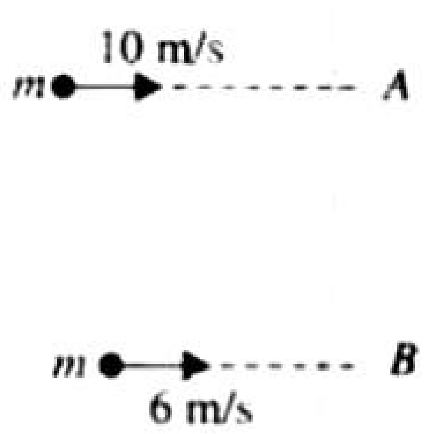
Subjective Type

1. A particle is projected at time $t = 0$ from a point P with a speed v_0 at an angle of 45° to the horizontal. Find the magnitude and the direction of the angular momentum of the particle about the point P at time $t = v_0/g$. **(IIT-JEE 1984)**
2. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance on the ground with respect to the point B (which is vertically below the end of the track as shown in fig.) where the sphere lands. During its flight as a projectile, does the sphere continue to rate about its centre of mass? Explain.



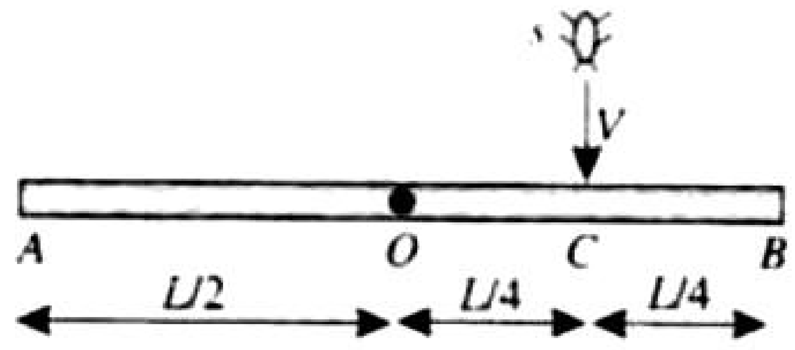
(IIT-JEE 1987)

3. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length $\sqrt{3}$ meters. Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m/s, and other with 6 m/s as shown in the figure. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Calculate the loss of the kinetic energy of the system in the above collision process. **(IIT-JEE 1989)**



4. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$. **(IIT-JEE 1990)**
5. A rigid body of radius of gyration k and radius R rolls (without slipping) down a plane inclined at an angle θ with horizontal. Calculate its acceleration and the frictional force acting on it. **(IIT-JEE 1991)**
6. A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (see figure). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the

insect moves towards the end B such that the rod rotates with a constant angular velocity ω

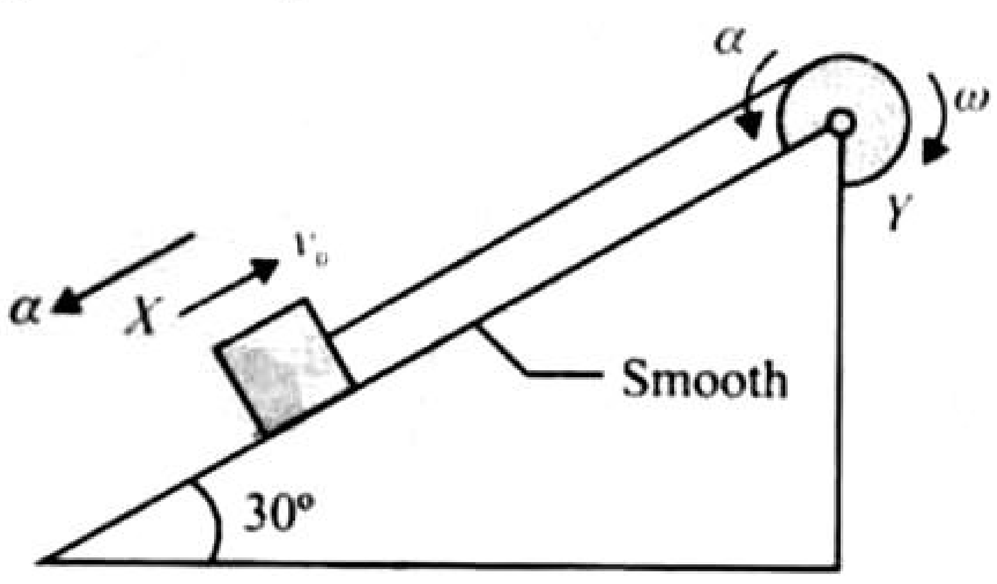


a. Determine the angular velocity w in terms of V and L.
b. If the insect reaches the end B when the rod has turned through an angle of 90°, determine V.

(IIT-JEE 1992)

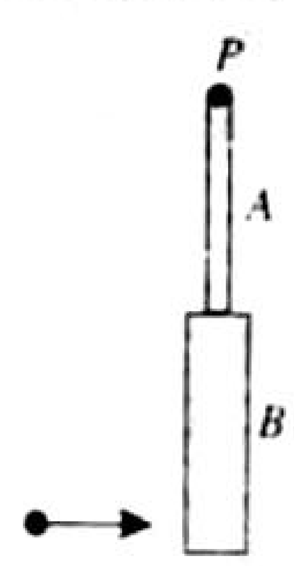
7. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30° to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in the figure. The drum is given an initial angular velocity such that block X starts moving up the plane.

a. Find the tension in the string during the motion.
b. At a certain instant of time, the magnitude of the angular velocity of Y is 10 rad/s. Calculate the distance travelled by X from that instant of time until it comes to rest. [$g = 10$ m/s^2]

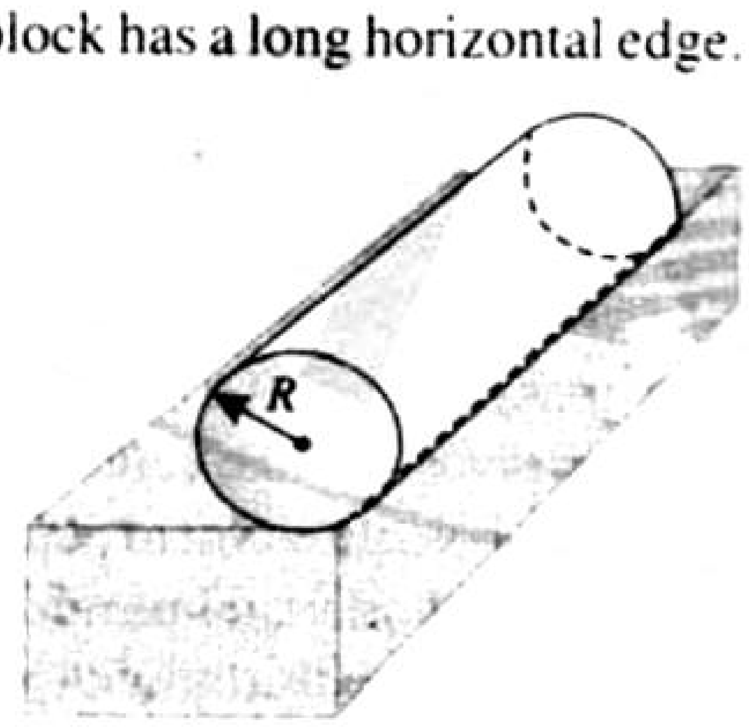


(IIT-JEE 1994)

8. Two uniform thin rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in the figure. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position. **(IIT-JEE 1994)**



9. A rectangular rigid fixed block has a long horizontal edge. A slid homogeneous cylinder of R is placed horizontally at rest its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical place as shown in the figure.

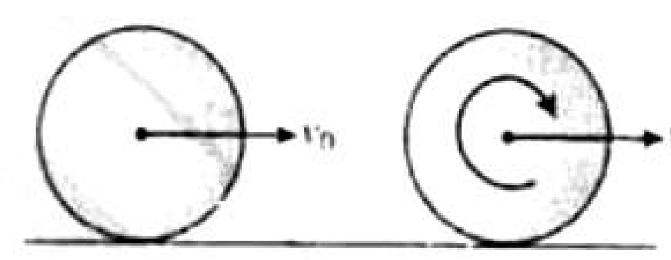


There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine: **(IIT-JEE 1995)**

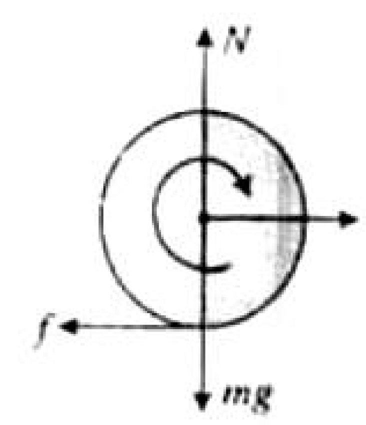
a. The angle θ through which the cylinder rotates before it leaves contact with the edge.
b. The speed of the cente of mass of the cylinder before leaving contact with the edge, and
c. The ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

10. A uniform disc of mass m and radius r is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at $t = 0$. At $t = t_0$ seconds it acquires a purely rolling motion.

a. Calculate the velocity of the centre of mass of the disc at $t = t_0$.
b. Assuming coefficient of friction to be μ calculate t_0.
c. The work done by the frictional force as a function of time
d. Total work done by the friction over a time t much longer than t_0. **(IIT-JEE 1997)**



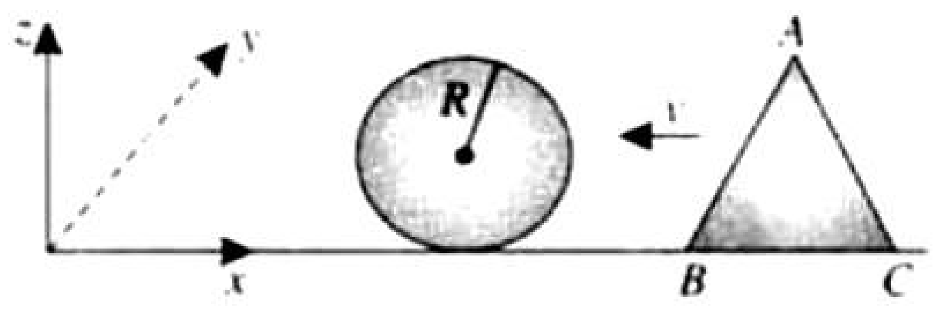
11. Two thin circular discs of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is perpendicular to the plane of the disc through their centres as shown in the figure. The object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough to prevent slipping. If the truck has an acceleration of 9 ms^{-2}, calculate



a. The force of friction on each disc.
b. The magnitude and direction of the frictional torque acting on each disc about the centre of mass O of the object. Take x-axis along the direction of the motion of the truck, and z-axis along vertically upwards direction. Express the torque in the vector form in terms of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ in the x, y and z direction respectively.
c. Find the minimum value of the coefficient of friction between the object and the floor of the truck which makes rolling of the object possible.

(IIT-JEE 1997)

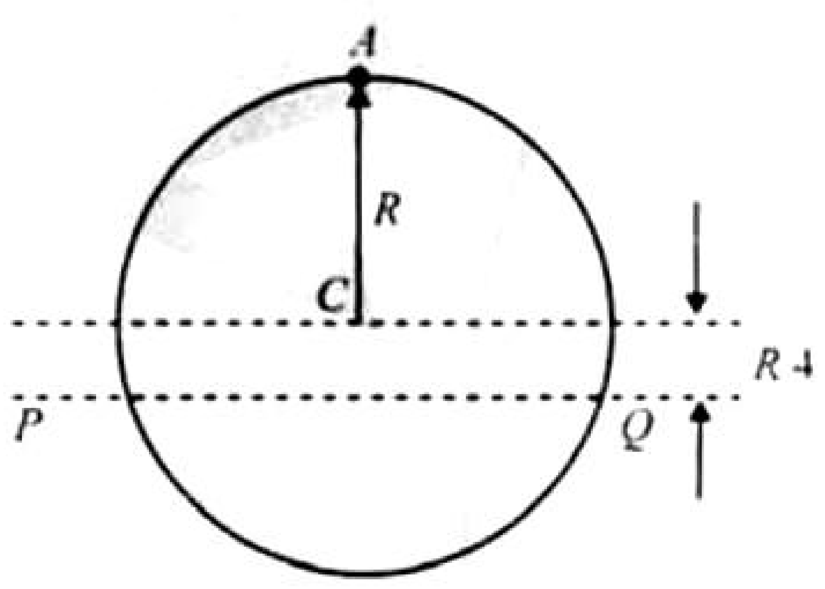
12. A wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time. Δt, during which the sphere exerts a constant force F on the wedge.



a. Find the force F and also the normal force N exerted by the table on the wedge during the time Δt.

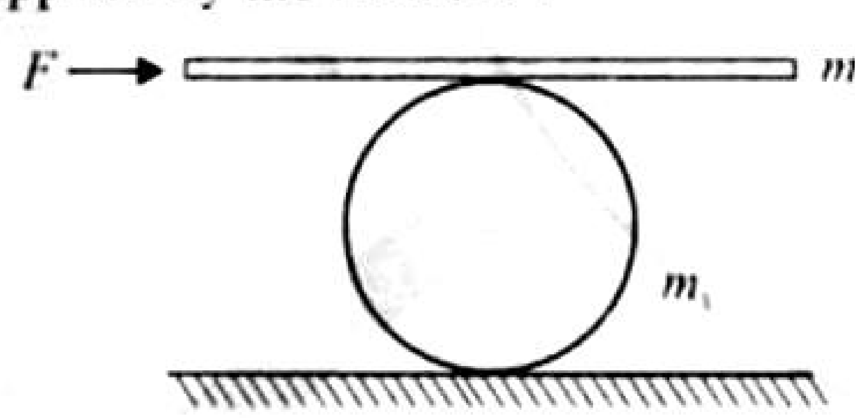
b. Let h denote the perpendicular distance between the centre of mass of the wedge and the line of action of F. Find the magnitude of the torque due to the normal force N about the centre of the wedge, during the interval Δt. **(IIT-JEE 1998)**

13. A uniform circular disc has radius R and mass m. A particle, also of mass m, is fixed at point A on the edge of the disc as shown in the figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially, the disc is held vertical with point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position.



(IIT-JEE 1998)

14. A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in the figure. There is no slipping at any contact. The horizontal component of the force applied by the man is F.



Find:

a. The acceleration of the plank and the centre of mass of the cylinder.

b. The magnitudes and directions of frictional forces at contact points. **(IIT-JEE 1999)**

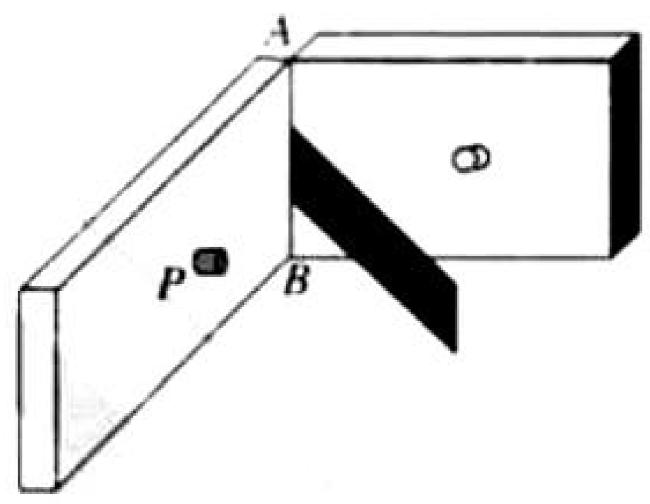
15. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end A of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

a. Find the ratio m/M.

b. A point P on the rod is at rest immediately after the collision. Find the distance AP.

c. Find the linear speed of a point P at time $\pi L/3v_0$ after the collision. **(IIT-JEE 2000)**

16. Two heavy metallic plates are joined together at t_0 90° to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its center of mass is 1.2 kg-m^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact.



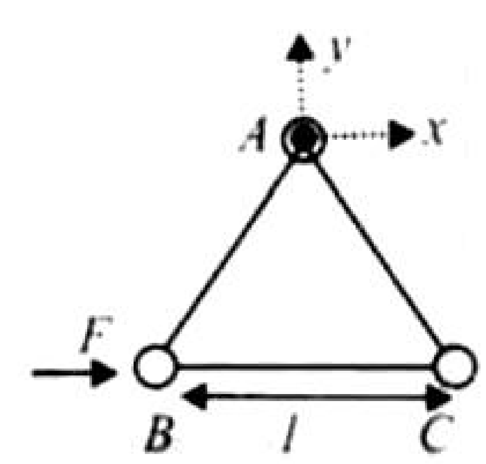
Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rad/s and turns back. If the impulse on the sheet due to each obstacle is 6 N-s,

a. find the location of the corner of mass of the laminar sheet from AB.

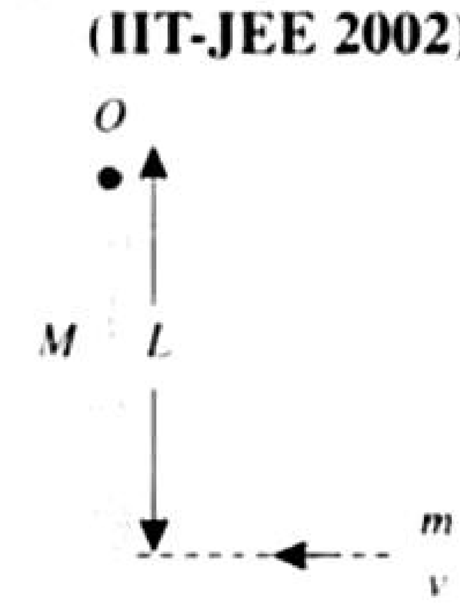
b. At what angular velocity does the laminar sheet come back after the first impact?

c. After how many impacts, does the laminar sheet come to rest? **(IIT-JEE 2001)**

17. Three particles A, B and C each of mass m are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged at point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about this axis with a constant angular velocity ω. (a) Find the magnitude of the horizontal force exerted by the hinge on the body. (b) At time T, when side BC is parallel to x-axis, force F is applied on B along BC (as in the figure). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T. **(IIT-JEE 2002)**

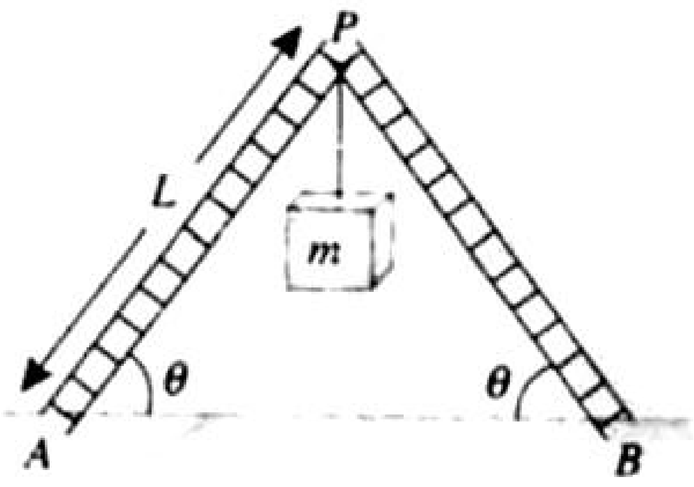


18. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O. **(IIT-JEE 2005)**



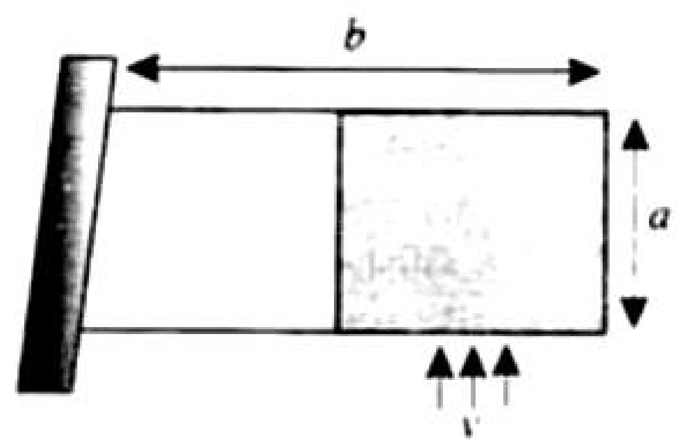
19. A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. **(IIT-JEE 2005)**

20. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.



(IIT-JEE 2005)

21. A rectangular plate of mass m and dimension $a \times b$ is held in horizontal position by striking n small balls (each of mass m) per unit area per second. The balls are striking in the shaded half region of the plate. The collision of the balls with the plate is elastic. What is v?



(Given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1 m; g = 10 m/s^2).

(IIT-JEE 2006)

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. c. **2.** c. **3.** b. **4.** c. **5.** c.
6. a. **7.** a. **8.** c. **9.** d. **10.** b.
11. a. **12.** b. **13.** b. **14.** a. **15.** a.
16. b. **17.** a. **18.** d. **19.** a. **20.** b.
21. b. **22.** d. **23.** a. **24.** b. **25.** b.
26. c. **27.** a. **28.** d. **29.** d.

Multiple Correct Answers Type

1. d. **2.** b. **3.** c.
4. b., d. **5.** a., b., c. **6.** a.
7. c. **8.** a. **9.** a., b, c.
10. c., d. **11.** a., d. **12.** b., c.
13. c. **14.** b., d. **15.** a., b.
16. c., d.

Linked Comprehension Type

1. c. **2.** a. **3.** b. **4.** d.
5. d. **6.** c. **7.** d. **8.** a.

Integer Answer Type

1. (4) **2.** (9) **3.** (3) **4.** (8) **5.** (2)
6. (4) **7.** (7) **8.** (6)

Assertion–Reasoning Type

1. d. **2.** d.

Fill in the Blanks Type

1. $\frac{2}{3}mg$ **2.** $\frac{Mw_0}{M+6m}$ **3.** $\frac{1}{2}MRAw^2$

4. $T \propto r^{-3}$ **5.** $\frac{mg}{6}$ **6.** $R_A = \left(\frac{d-x}{d}\right)w$

7. $48\,Ma^2$

True/False Type

1. False **2.** False **3.** False **4.** False

Subjective Type

1. $\frac{mv_0^3}{2\sqrt{2}g}$ in a direction perpendicular to paper inwards.

2. 2.13 m, yes **3.** 2.72 J

4. $v = \sqrt{\frac{14Rg}{3}}$ **5.** $\mu_s \geq \frac{\tan\theta}{1+\frac{R^2}{k^2}}$

This is the condition on μ_s so that the body rolls without slipping.

6. (a) $\frac{12V}{7L}$ (b) 3.5 ms^{-1}

7. (a) 1.63 N (b) 1.22 m

8. 6.3 m/s

9. (a) $\cos\theta = \frac{4}{7}$ (b) $\sqrt{\frac{4gR}{7}}$ (c) 6

10. (a) $\frac{2}{3}v_0$

(b) $\frac{v_0}{3\mu g}$, For $t \leq t_0$, $W_f = \frac{m\mu g t}{2}[3\mu g t - 2v_0]$, $\frac{-mv_0^2}{6}$

11. (a) $6\hat{i}$ N (b) $-0.6\hat{j}-0.6\hat{k}$ (c) $\mu \geq \frac{a}{3g}$

12. (a) (i) $\vec{F}=\left(\frac{2mv}{\Delta t}\right)\hat{i}-\left(\frac{2mv}{\sqrt{3}\Delta t}\right)\hat{k}$

(ii) $\vec{N}=\left(mg+\frac{2mv}{\sqrt{3}\Delta t}\right)\hat{k}$

(b) $|\vec{\tau}_N|=\left(\frac{4mv}{\sqrt{3}\Delta t}\right)h$

13. $\sqrt{5gR}$

14. (a) $a_{CM}=\frac{4F}{3m_1+8m_2}, a_{\text{plank}}=\frac{8F}{3m_1+8m_2}=2a_{CM}$

(b) $\frac{3Fm_1}{3m_1+8m_2}, \frac{Fm_1}{3m_1+8m_2}$

15. (a) $\frac{1}{4}$ (b) $\frac{2L}{3}$ (c) $\frac{v_0}{2\sqrt{2}}$

16. (a) 0.1 m (b) 1 rad/s (c) sheet will never come to rest.

17. (a) $\sqrt{3}ml\omega^2$ (b) $(F_{\text{net}})_x=\frac{-F}{4}, (F_{\text{net}})_y=\sqrt{3}\,ml\omega^2$

18. $\frac{3mv}{(M+3m)L}$

19. $\frac{2}{3}g\sin\theta$

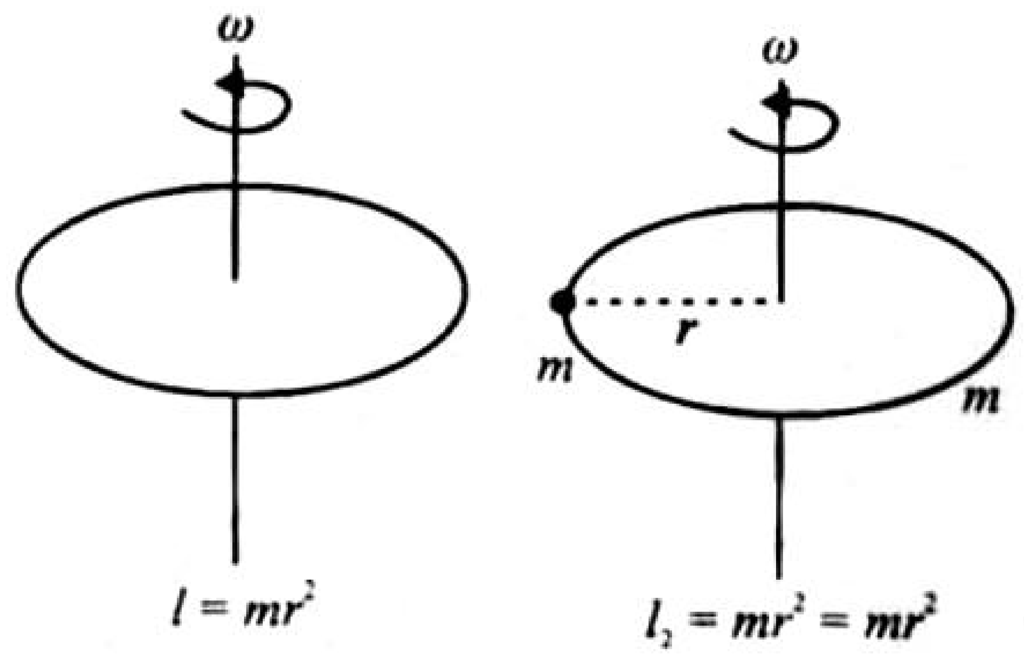
20. $\left[(M+m)\frac{g}{2}\right]\cot\theta$, the direction of friction force at A is towards right and at B towards left.

21. 10 m/s

JEE Advanced

Single Correct Answer Type

1. c. Since the objects are placed gently, therefore no external torque is acting on the system. Therefore, angular momentum is constant.



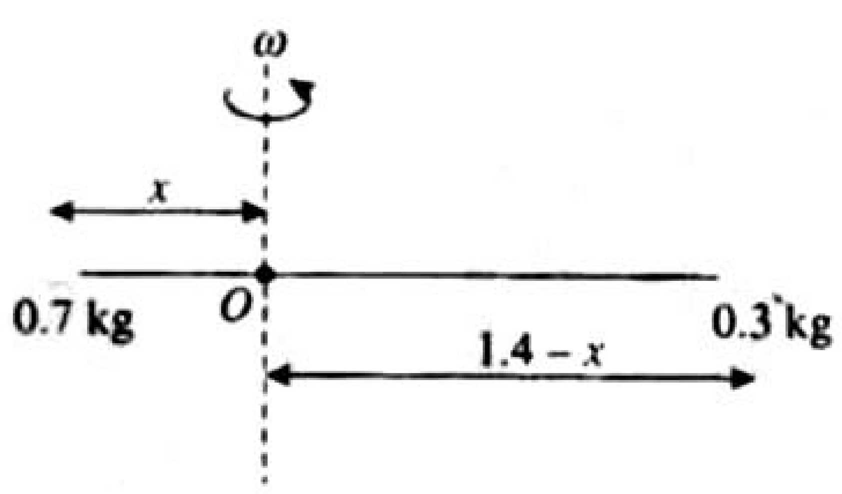
i.e., $I_1\omega_1 = I_2\omega_2$

$$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\omega_2$$

$$\therefore \quad \omega_2 = \frac{M\omega}{M+2m}$$

2. c. The moment of inertia of the system about axis of rotation O is

$$I = I_1 + I_2$$
$$= 0.3x^2 + 0.7(1.4 - x)^2$$
$$= 0.3x^2 + 0.7(1.96 + x^2 - 2.8x)$$
$$= x^2 + 1.372 - 1.96x$$



The work done in rotating the rod is converted into its rotational kinetic energy.

$$\therefore \quad W = \frac{1}{2}I\omega^2 = \frac{1}{2}[x^2 + 1.372 - 1.96x]\omega^2$$

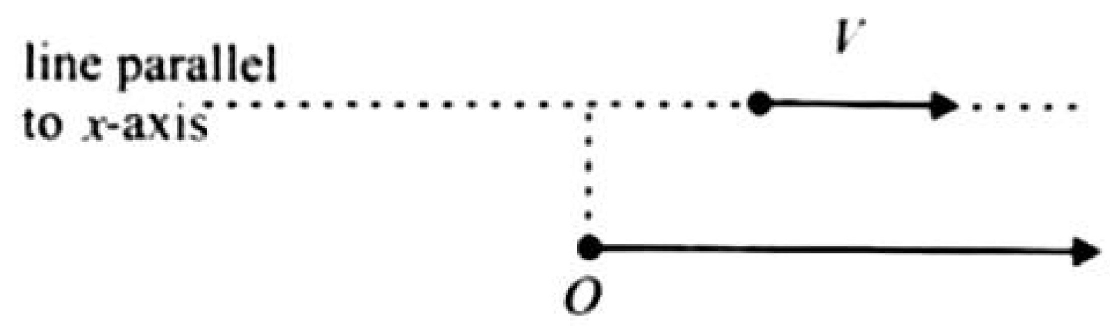
For work done to be minimum

$$\frac{d\omega}{dx} = 0$$

$$\Rightarrow \quad 2x - 1.96 = 0$$

$$\Rightarrow \quad x = \frac{1.96}{2} = 0.98 \text{ m}$$

3. b. Angular momentum of mass m moving with a constant velocity about origin is

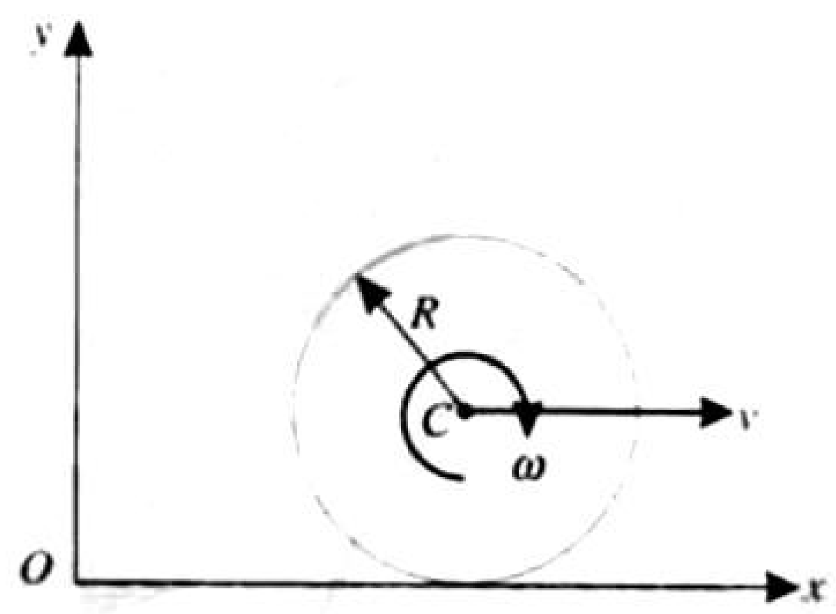


L = momentum × perpendicular distance of line of action of momentum from origin, $L = mv \times y$

In the given condition, mvy is a constant. Therefore, angular momentum is constant.

4. c. As the spheres are smooth, there will be no transfer of angular momentum. Thus, A, after collision, will remain with its initial angular momentum.

5. c. The disc has two types of motion, namely, translational and rotational. Therefore, there are two types of angular momentum and the total angular momentum is the sum of these two.



$L = L_T + L_R$, L_T = angular momentum due to translational motion

L_R = angular momentum due to rotational motion about CM

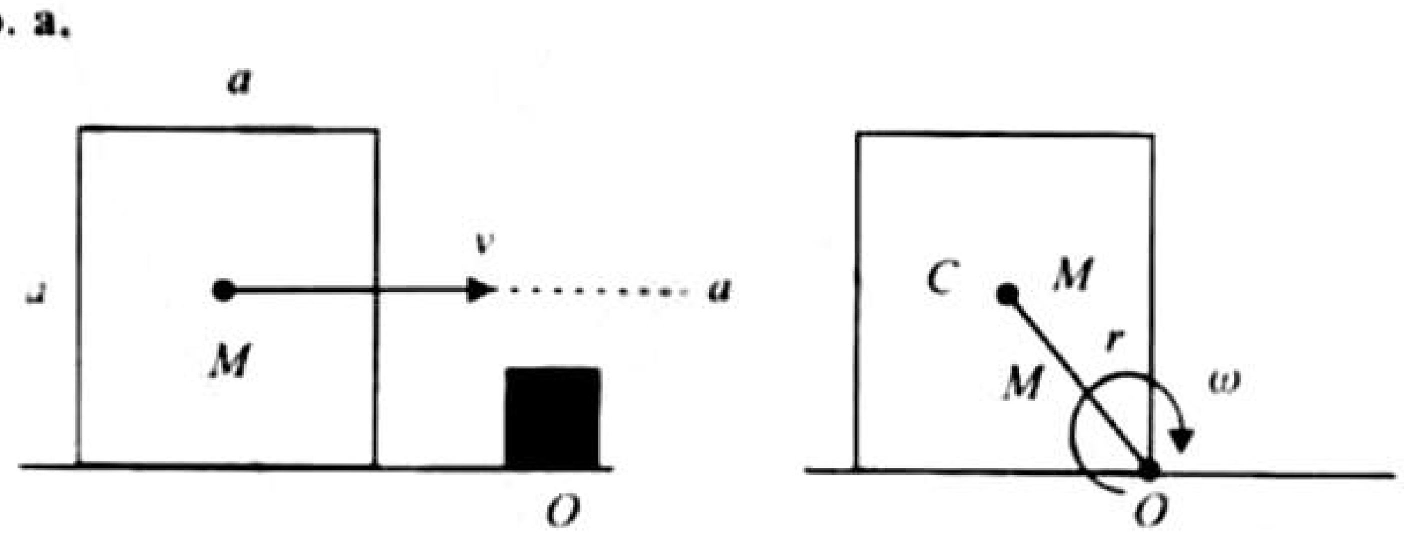
$$L = MV \times R + I_{CM}\,\omega$$

I_{CM} = MI about centre of mass C

$\Rightarrow$ $L = M(R\,\omega)R + \frac{1}{2}MR^2\omega$ ($V = R\,\omega$ in case of rolling motion and surface at rest)

$= \frac{3}{2}MR^2\omega$

6. a.



$r = \sqrt{2}\,\frac{a}{2}$ or $r^2 = \sqrt{2}\,\frac{a^2}{2}$

Net torque about O is zero.

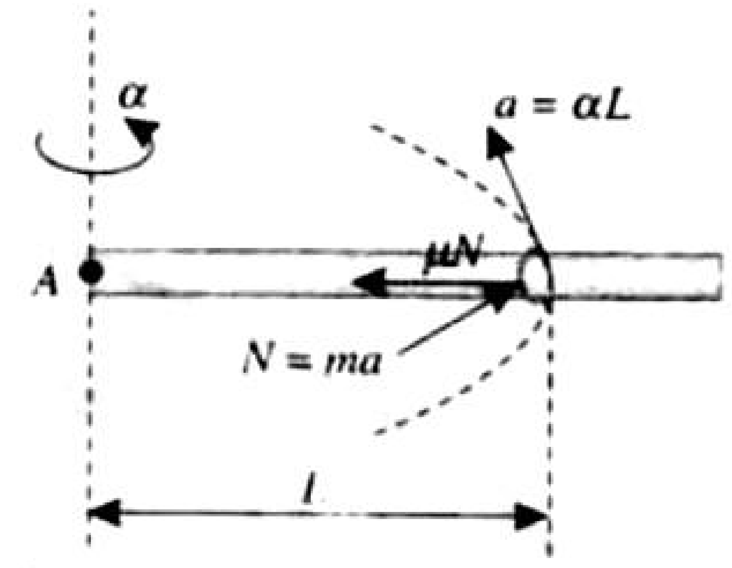
Therefore, angular momentum (L) about O will be conserved, or $L_i = L_f$

$$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{CM} + Mr^2)\omega$$

$$= \left(\frac{Ma^2}{6} + M\frac{a^2}{2}\right)\omega$$

$$\omega = \frac{3V}{4a}$$

7. a. When we are giving an angular acceleration to the rod, the bead is also having an instantaneous acceleration $a = L\alpha$. This will happen when a force is exerted on the bead by the rod. The bead has a tendency to move away from the centre. But due to friction between the bead and the rod, this does not happen to the extent to which frictional force is capable of holding the bead. The frictional force here provides the necessary centripetal force. If the instantaneous angular velocity is ω, then



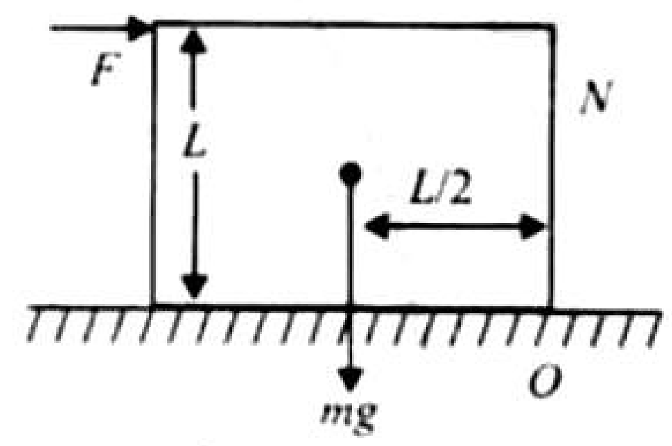
$$mL\omega^2 = \mu(ma) = \mu mL\alpha$$

$\Rightarrow$ $\omega^2 = \mu\alpha$

By applying $\omega = \omega_0 + \alpha t$, we get $\omega = \alpha t$

$\therefore$ $\alpha^2 t^2 = \mu\alpha \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$

8. c. The applied force shifts the normal reaction to one corner as shown in the figure. At this situation, the cubical block starts topping about O. Taking torque about O, we get



$$F \times L = mg \times \frac{L}{2}$$

$\Rightarrow$ $F = \frac{mg}{2}$

9. d. The moment of inertia of the loop about XX' axis is

$$I_{XX'} = \frac{mR^2}{2} + mR^2 + \frac{3}{2}mR^2$$

where m = mass of the loop and R = radius of the loop

Here $m = L\rho$ and $R = \frac{L}{2\pi}$; therefore,

$$I_{XX'} = \frac{3}{2}(L\rho)\left(\frac{L}{2\pi}\right)^2 = \frac{3L^3\rho}{8\pi^2}$$

10. b. The MI about the axis of rotation is not constant as the perpendicular distance of the bead with the axis of rotation increases.

Also since no external torque is acting, therefore

$$\tau_{CM} = \frac{dL}{dt}$$

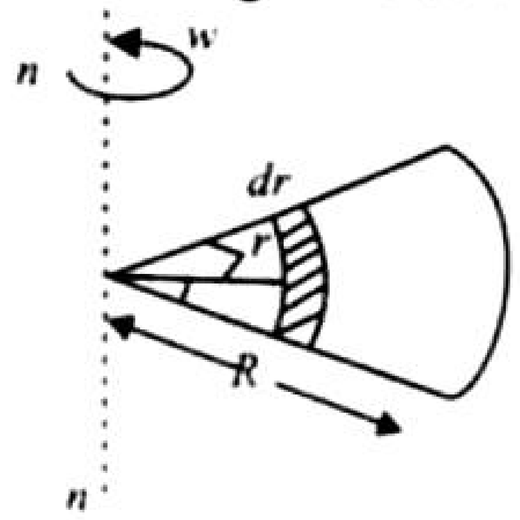
$\Rightarrow$ L = constant

$I\omega$ = constant

Since I increases, ω decreases.

11. a. We cannot consider the quadrant as a single mass as the distance of different particles is different from the axis of rotation. So we take the help of calculus. Let us consider a segment as shown in the figure. All masses lying in this segment are at a distance r

from the axis and hence considered as a small differential mass dm. Let the thickness of the segment be dr.



The mass per unit area of the quadrant $= \frac{M}{\pi R^2/4} = \frac{4M}{\pi R^2}$

Area of the segment $= \frac{1}{4}[\pi(r+dr)^2 - \pi r^2] = \frac{1}{4} \times 2\pi r dr$

$= \frac{\pi r dr}{2}$

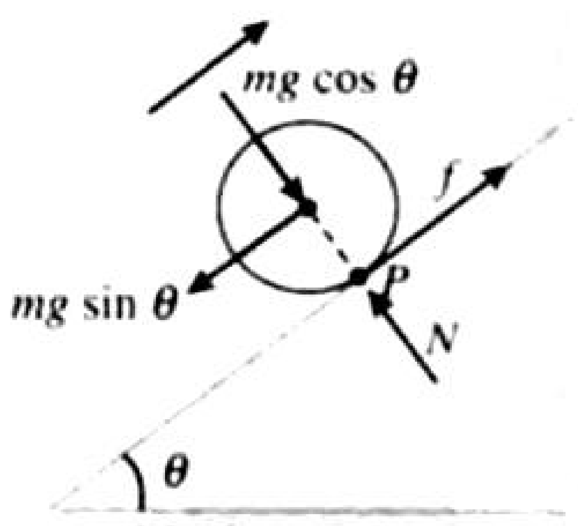
Mass of the segment $dm = \frac{\pi r dr}{2} \times \frac{4M}{\pi R^2} = \frac{2M}{R^2} r dr$

MI of this segment about $nn' = \frac{2M}{R^2} r dr \times r^2 \frac{2M}{R^2} r^3 dr$

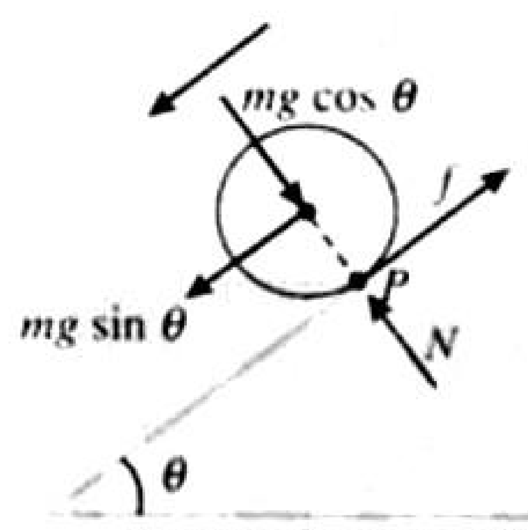
MI of the quadrant about $nn' = \int_0^R \frac{2M}{R^2} r^2 dr$

$= \frac{2M}{R^2} \times \frac{R^4}{4} \times \frac{MR^2}{2}$

12. b. The frictional force acts in a direction opposite to the direction of net acceleration of point of contact. In both cases the cylinder rolls up or rolls down, the point of contact P has an acceleration down the plane, similar to that of center of mass of cylinder.



Moving up the plane

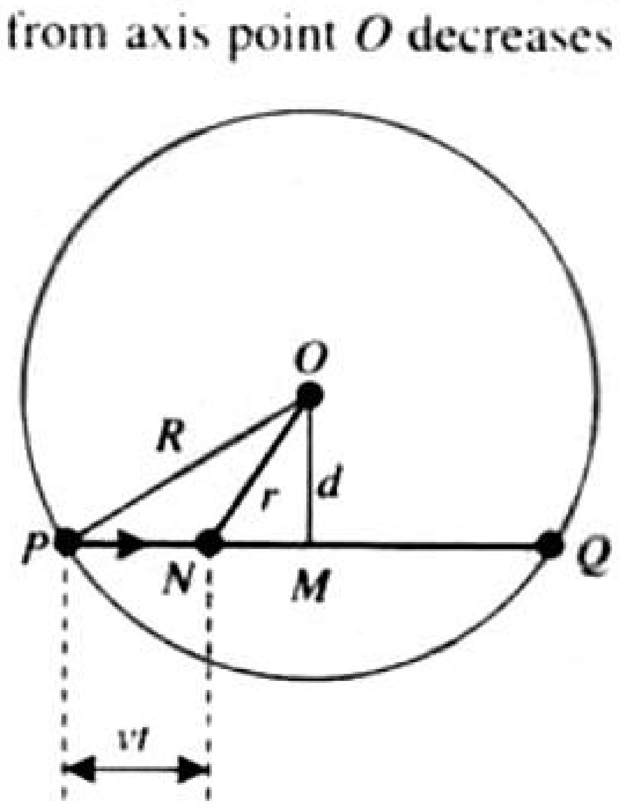


Moving down the plane

∴ The frictional force, acting on the cylinder, is always up the incline, while ascending as well as descending.

13. b. In absence of external torque, hence according to the conservation of angular momentum the angular momentum of the system remains conserved.

Let the tortoise move along the chord PQ. When the tortoise moves from P and M its distance from axis point O decreases and so the moment of inertia decreases as $I = mr^2$, where r = distance of tortoise from O.



When the tortoise moves from M to Q the distance r increases and so I also increases.

But according to the conservation of angular momentum the angular momentum of the system remains conserved $I\omega$ = constant.

$$I_P = mR^2 + \frac{MR^2}{2} \quad \text{(i)}$$

$$I_N = mr^2 + \frac{MR^2}{2} \quad \text{(ii)}$$

By geometry, $r^2 = d^2 + \left[\sqrt{R^2 - d^2} - vt\right]^2$ (iii)

Angular momentum is conserved

$$I_P\omega_0 = I_N\omega(t) \quad \text{or} \quad \omega(t) = \frac{I_A\omega_0}{I_B} \quad \text{(iv)}$$

$\omega(t)$ depends on I_N, I_N depends on r and r depends on time t. The function of t is non-linear.

Hence $\omega(t)$ is a non-linear function of t.

ω increases when tortoise travels from P to M as momentum of inertia of the system decreases and ω decrease when tortoise travels from P to Q as during this interval momentum of inertia of the system increases.

First increase and then decrease of ω is revealed in graphs (b) and (d). But $\omega(t)$ is a non-linear function of t, hence graph (b) represents the variation of $\omega(t)$ w.r.t time.

14. a. Change in angular momentum of the system = angular impulse given to the system (angular momentum)$_f$ – (angular momentum)$_i$

$= MV \times \frac{L}{2}$ (i)

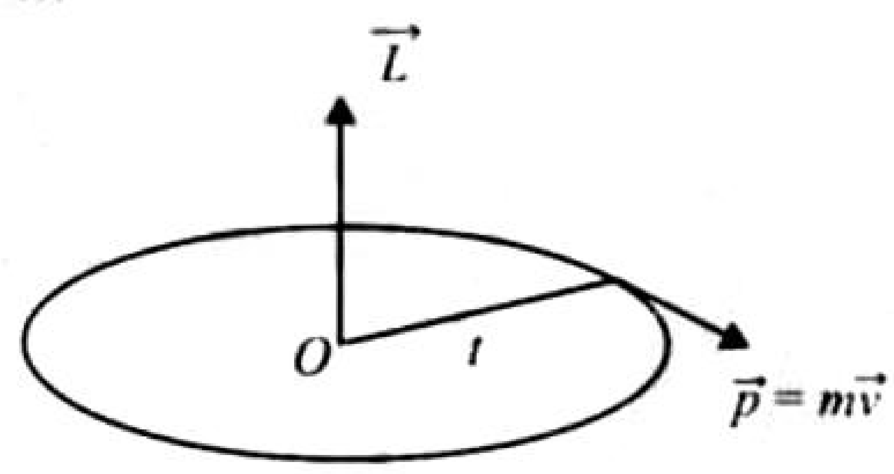
Let the system starts rotating with the angular velocity ω. Moment of inertia of the system about its axis of rotation (centre of mass of the system) $= M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2 = \frac{2ML^2}{4}$

$= \frac{ML^2}{2}$ (From (i))

$I\omega - 0 = MV \times \frac{L}{2}$

$\Rightarrow \quad \omega = \frac{MV}{I} \times \frac{L}{2} = \frac{MV}{ML^2/2} \times \frac{L}{2} = \frac{V}{L}$

15. a. $|\vec{L}| = mvr$



The direction of $\vec{L}$ (about the centre) is perpendicular to the plane containing the circular path. Both magnitude and direction of the angular momentum of the particle moving in a circular path about its centre O are constant.

16. b. The linear momentum (L) is conserved, since τ_{ext} is zero.

Let $I_1 = I_0$, therefore $I_f = 2I_0$

$\omega_1 = \omega_0$, therefore $\omega_f = \omega$ (say)

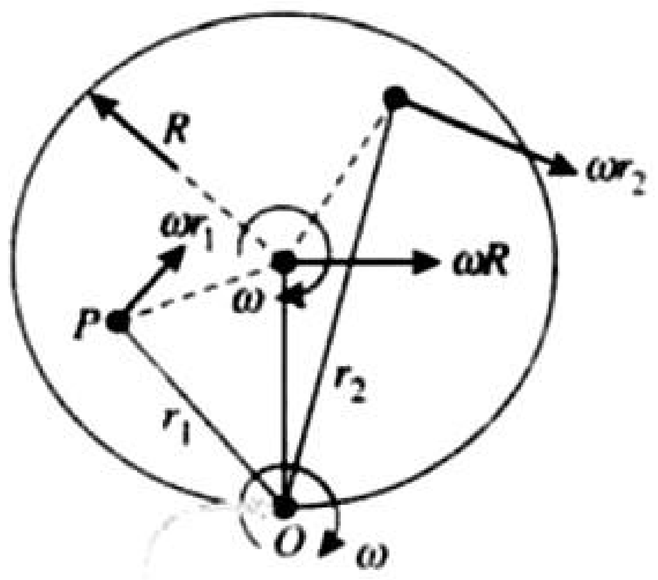
∴ $I_0\omega_0 = 2I_0\omega \Rightarrow \omega = \frac{\omega_0}{2}$

$K = \frac{1}{2}I\omega_0^2$

$K_f = \frac{1}{2}(2I_0) \times \left(\frac{\omega_0}{2}\right)^2$

$= \frac{1}{2} \times 2I_0 \times \frac{\omega_0^2}{4}$

17. a. The rolling disc can be considered as pure rotation about point of contact O. In this case point O will act as instantaneous center of rotation of disc.

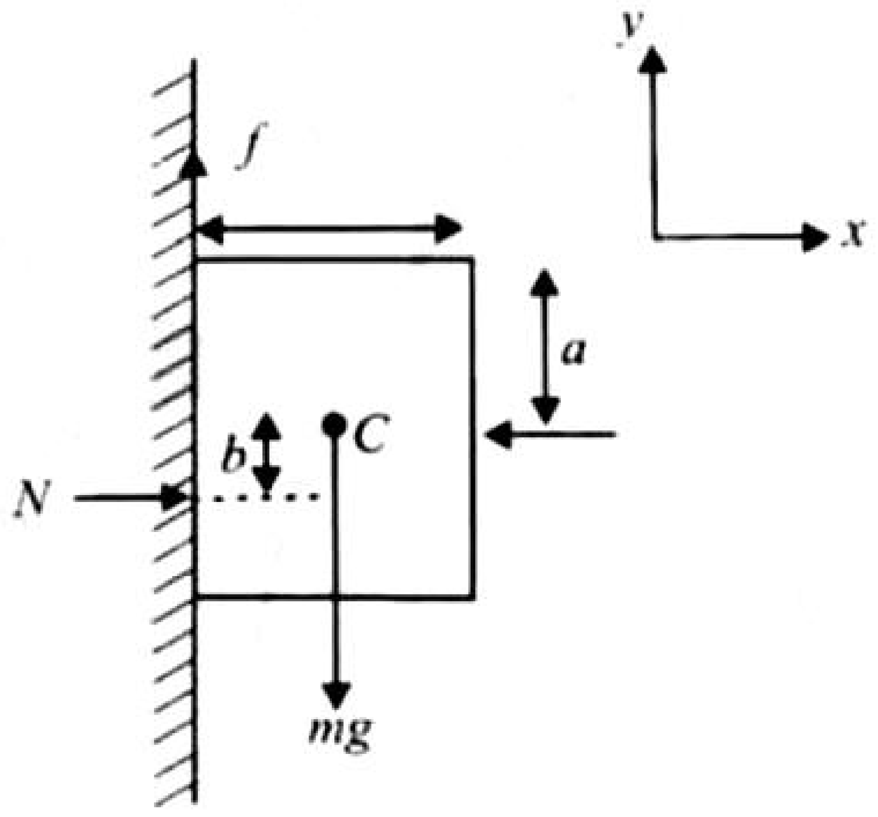


Instantaneous center of rotation

Hence velocity of point P, $v_P = \omega r_1$
velocity of point Q, $v_Q = \omega r_2$
velocity of point C, $v_C = \omega R$
for diagram it is clear

$$r_2 > R > r_1 \Rightarrow v_Q > v_C > v_P$$

18. d. The cubical block is in equilibrium. For translational equilibrium,



a. $\Sigma F_x = 0 \Rightarrow F = N$

b. $\Sigma F_y = 0 \Rightarrow f = mg$

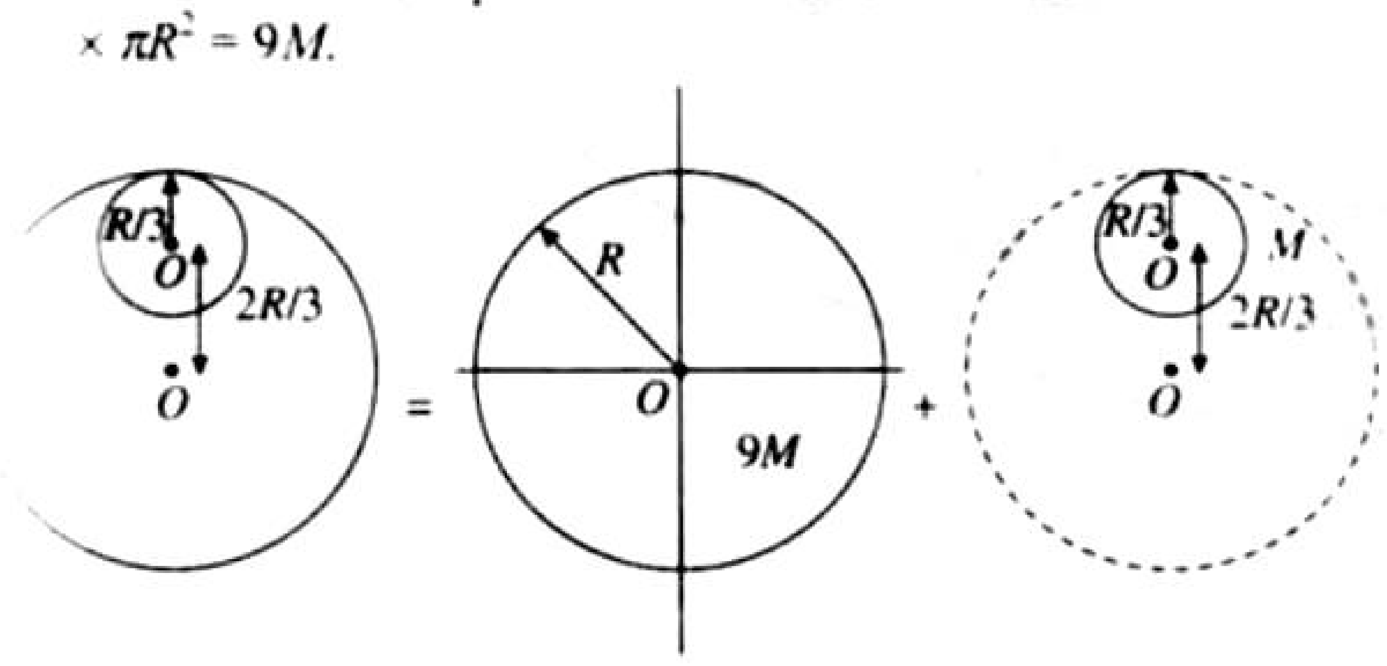
For rotational equilibrium,

$\Sigma\tau_c = 0$, where τ_c = torque about CM C

Torque is created by frictional force (f) about C = $\tau_c = f \times a$ in a clockwise direction. There should be another torque which should counter this torque. The normal reaction N on the block acts as shown in the figure. This will create a torque $N \times b$ in the anticlockwise direction. Such that $f \times a = N \times b$

Note: The normal force does not act through the centre of the body always. The point of application of the normal force depends on all the forces acting on the body.

19. a. Let σ be the mass per unit area. The total mass of the disc = σ × $\pi R^2 = 9M$.



M = Mass of the circular disc cut

$$= \sigma \times \pi\left(\frac{R}{3}\right)^2$$

$$= \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M superimposed on it.

Moment of inertia (I_1) of the complete disc = $9MR^2/2$ about an axis passing through O and perpendicular to the plane of the disc.

MI of the cutout portion about an axis passing through O' and perpendicular to the plane of disc is

$$\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

Therefore, MI (I_2) of the cutout portion about an axis passing through O and perpendicular to the plane of disc is

$$\left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

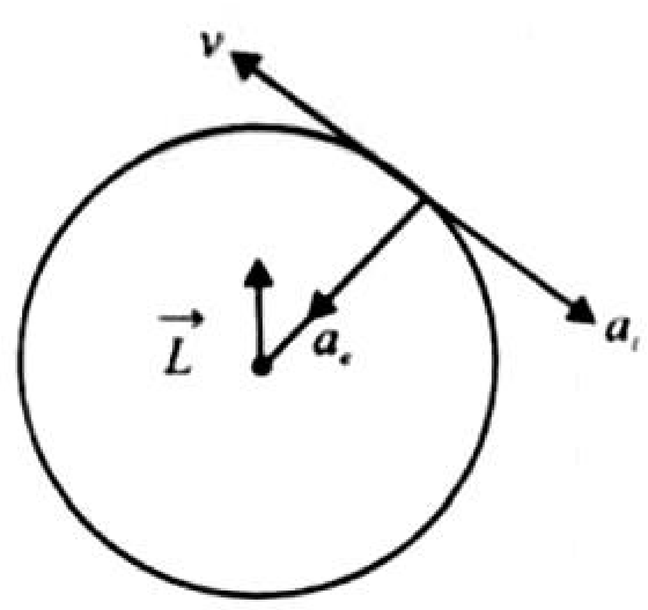
Therefore, the total MI of the system about an axis passing through O and perpendicular to the plane of the disc is

$$I = I_1 + I_2$$

$$= \frac{1}{2}\,9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]^2$$

$$= \frac{1}{2}\,9MR^2 - MR^2\left[\frac{1}{18} + \frac{4}{9}\right]$$

20. b. $|\vec{L}| = mvr$. Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it cannot have spiral path. Since the particle has two accelerations a_c and a_t, therefore the net acceleration is not towards the centre.



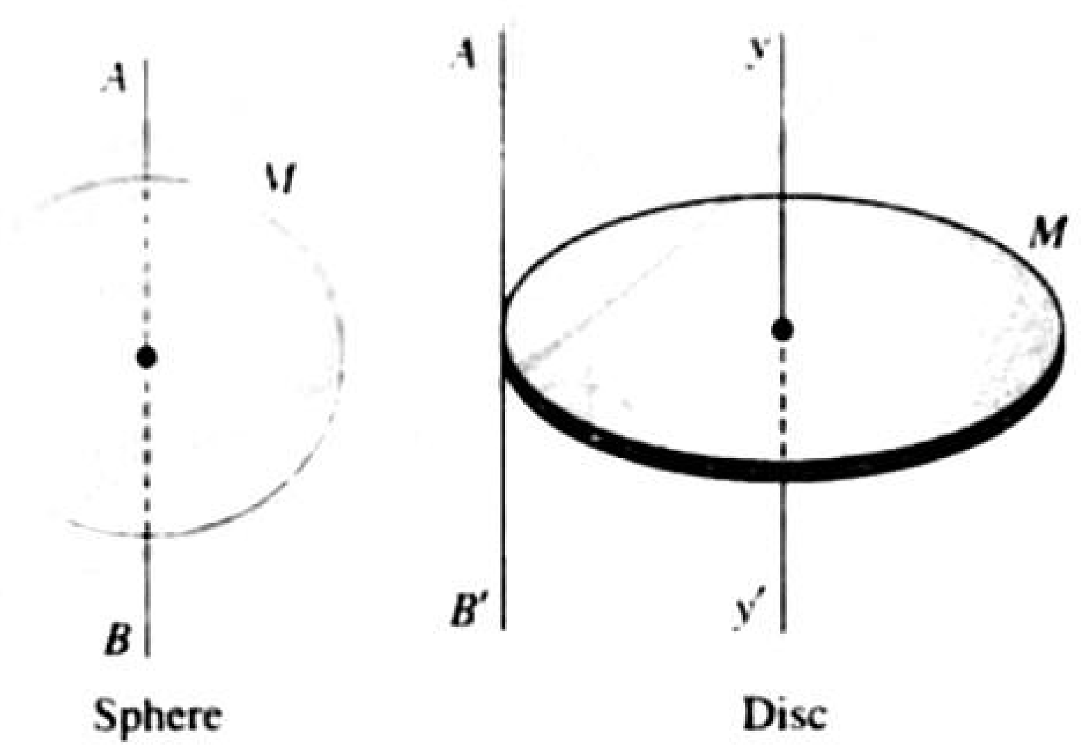
The direction of $\vec{L}$ remains the same even when the speed decreases.

21. b. $I_{AB} = \frac{2}{5}MR^2 = I$ (given) (i)

$$I_{A'B'} = I_{YY'} + Mr^2 = I\,\frac{1}{2}Mr^2 + Mr^2$$

$$= \frac{3}{2}Mr^2 = 1 \text{ (given)} \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\frac{2}{5}MR^2 = \frac{3}{2}Mr^2 \Rightarrow r = \frac{2}{\sqrt{15}}R$



22. d. By the concept of energy conservation,

$$\frac{1}{2}mV^2 + \frac{1}{2}I\frac{V^2}{R^2} = \frac{3}{4}mV^2$$

$$\therefore \quad \frac{1}{2}I\frac{V^2}{R^2} = \frac{3}{4}mV^2 - \frac{1}{2}mV^2 = \frac{1}{4}mV^2$$

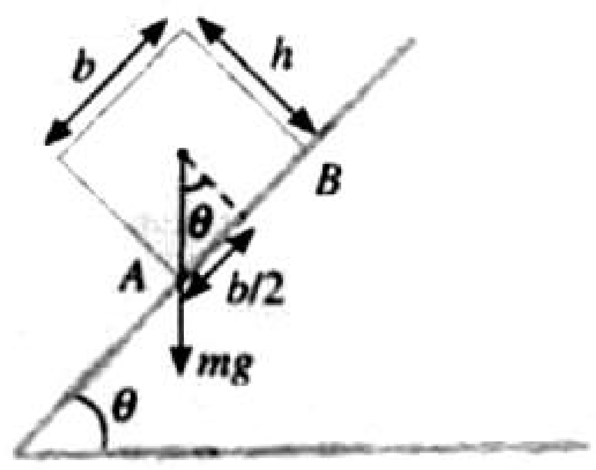
$$\Rightarrow \quad I = \frac{1}{2}mR^2$$

23. a. Linear momentum remains constant if net external force on the system of particles is zero.

24. b. The maximum angle when the block will not slide should be equal to the angle of repose.

$$\phi = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$$

If the block does not topple, the line of action of weight should pass within the base of the block



And for toppling, maximum angle of inclination,

$$\theta = \tan^{-1}\frac{a/2}{h/2}$$

$$\theta = \tan^{-1}\frac{(10/2)}{(15/2)} = \tan^{-1}\frac{2}{3}$$

$$\theta = 34° < 60°$$

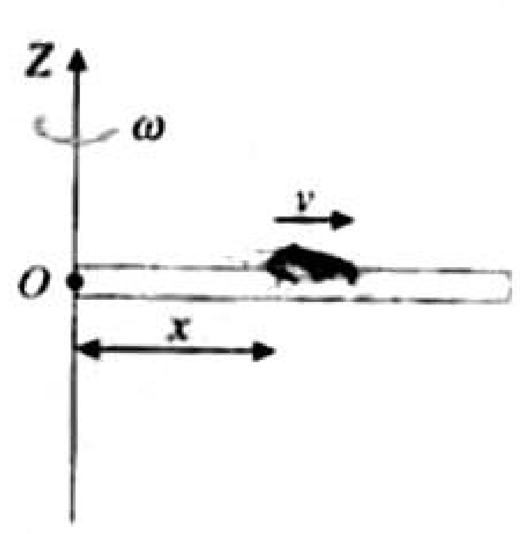
For 60°, mg is already outside the base.

$\therefore$ As the angle θ is increased first it will topple before it starts sliding.

25. b. Let M and l be the mass and length of the rod respectively and m be the mass of the insect. Let insect be at a distance x from O at any instant of time t.

$$\therefore \quad x = vt \qquad \text{(i)}$$

Angular momentum of the system about O,



$$L = \left[\frac{Ml^2}{3} + mx^2\right]\omega$$

$$= \left[\frac{Ml^2}{3} + m(vt)^2\right]\omega \qquad \text{(using (i))}$$

$$= \left[\frac{Ml^2}{3} + mv^2t^2\right]\omega$$

As $|\vec{\tau}| = \frac{dL}{dt} = \frac{d}{dt}\left[\frac{Ml^2}{3} + mv^2t^2\right]\omega$

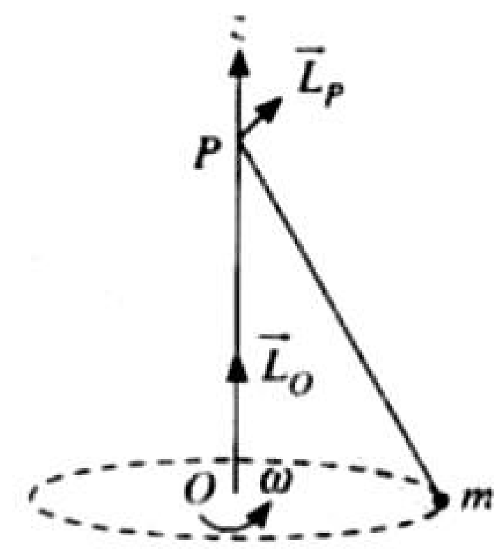
As ω and v remain constant

$$\therefore \quad |\vec{\tau}| = 2m\omega v^2 t$$

$$|\vec{\tau}| \propto t$$

Hence, the graph and t is a straight line passing through the (0, 0). Option (b) represents correct plot.

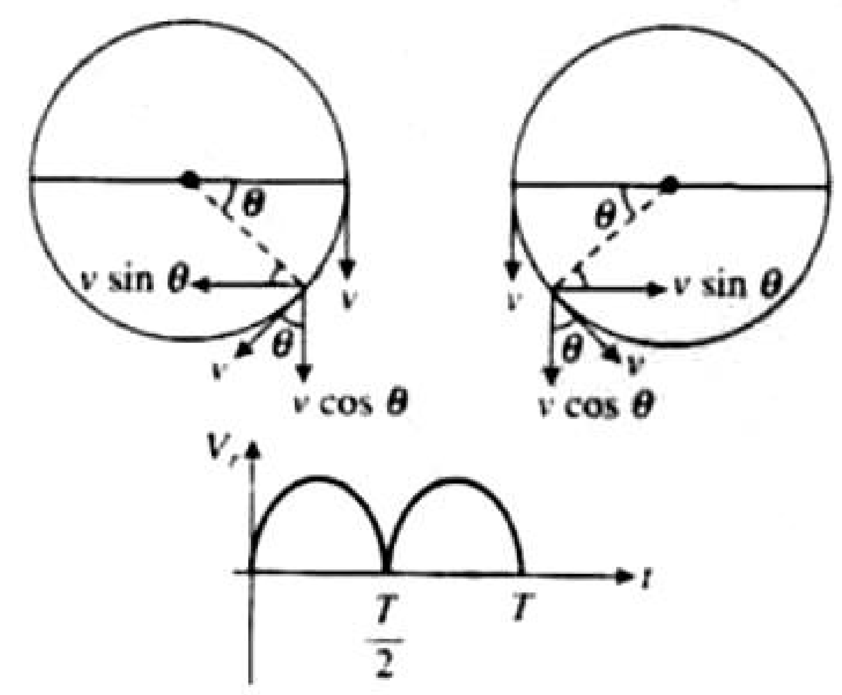
26. c.



Magnitude and direction of $\vec{L}_O$ remain constant.

Magnitude of $\vec{L}_P$ remains constant but direction of $\vec{L}_P$ change.

27. a.



$V_r = |2v \sin\theta| = |2v \sin\omega t|$

28. d. $I_P > I_Q$

$$a_P = \frac{g\sin\theta}{1 + \frac{I_P}{mR^2}}, \quad a_Q = \frac{g\sin\theta}{1 + \frac{I_Q}{mR^2}}$$

$$a_p < a_Q \Rightarrow V = u + at \Rightarrow t \propto \frac{1}{a}$$

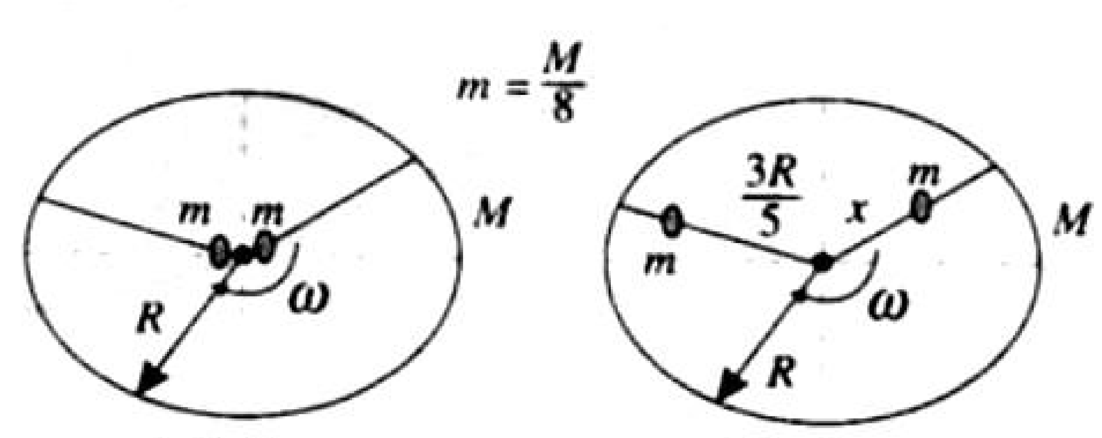
$$t_P > t_Q$$

$$V^2 = u^2 + 2as \Rightarrow v \propto a \Rightarrow V_P < V_Q$$

Translational KE, $\frac{1}{2}mV^2 \Rightarrow$ TR $KE_P <$ TR KE_Q

$$V = \omega R \Rightarrow \omega \propto V \Rightarrow \omega_P < \omega_Q$$

29. d. Angular momentum of ring and mass system should be conserved



Initially Finally

$I\omega$ = constant

Initial moment of inertia of system (ring + masses)

$I_{initial} = MR^2$

Final moment of inertia

$$MR^2 \cdot \omega = \left(MR^2 + \frac{M}{8}\frac{9}{25}R^2 + \frac{M}{8}x^2\right)\frac{8\omega}{9}$$

$$\frac{9}{8}MR^2 = MR^2 + \frac{9}{25}\frac{MR^2}{8} + \frac{M}{8}x^2$$

$$\frac{Mx^2}{8} = MR^2\left(\frac{1}{8} - \frac{9}{25 \times 8}\right)$$

$$\frac{M}{8}x^2 = \frac{2}{25}MR^2$$

$$\Rightarrow \quad x^2 = \frac{16}{25}R^2 \Rightarrow x = \left(\frac{4}{5}\right)R$$

Multiple Correct Answers Type

1. d. We know that $F_{ext} = Ma_{CM}$ (i)

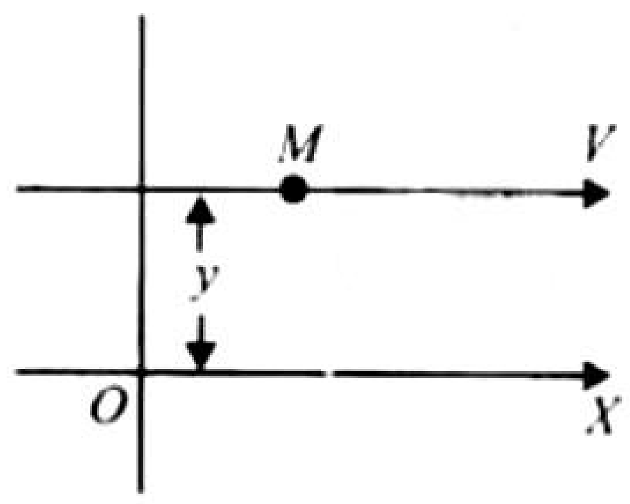
We consider the two particles in a system. Mutual force of attraction is an internal force. There are no external forces acting on the system. From Eq. (i), we get

$a_{CM} = 0$

Since initial $v_{CM} = 0$, therefore, final $v_{CM} = 0$

2. b. Angular momentum

$\vec{L} = \vec{r} \times \vec{p}$



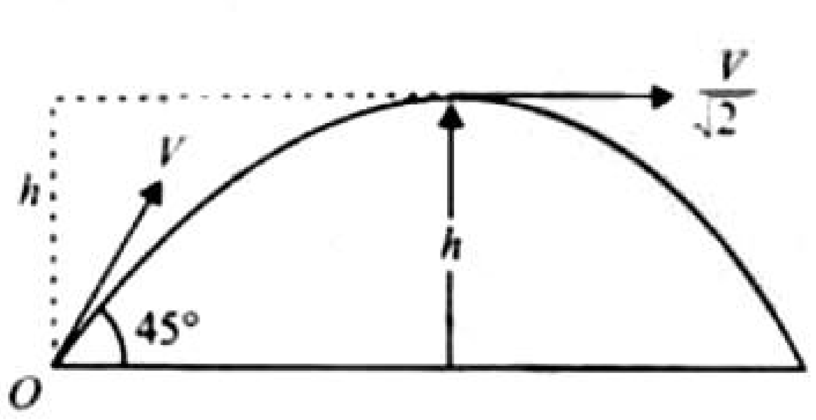
L = Momentum × perpendicular distance of line of action of momentum w.r.t. point of rotation

$L = MV \times y$

The quantities on the right side of the equation are not changing. Thus, magnitude is constant.

3. c. When the cycle is not pedaled but it is in motion (due to previous effort) the wheels move in the direction such that the centre of mass of the wheel moves forward. Rolling friction will act in the opposite direction to the relative motion of the centre of mass of the body with respect to the ground. Therefore the rolling friction will act in the backward direction in both the wheels. The sliding friction will act in the forward direction of the rear wheel during pedaling.

4. b, d. Angular momentum = (momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)



Angular momentum about O, $L = \frac{mv}{\sqrt{2}} \times h$ (i)

Now, $h = \frac{V^2 \sin^2\theta}{2g} = \frac{V^2}{4g}$ [$\because \theta = 45°$] (ii)

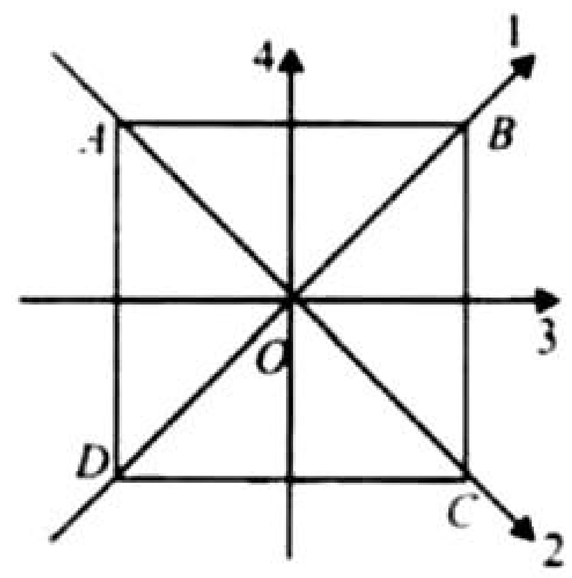
From Eqs. (i) and (ii), we get

$$L = \frac{mv}{\sqrt{2}}(2\sqrt{gh})h = m\sqrt{2gh^3}$$

Also from Eqs. (i) and (ii), we get

$$L = \frac{mV}{\sqrt{2}} \times \frac{V^2}{4g} = \frac{mV^3}{4\sqrt{2}\,g}$$

5. a., b., c. To find the moment of inertia of $ABCD$ about an axis passing through centre O and perpendicular to the plane of the plate, we use perpendicular axis theorem. If we consider $ABCD$ to be in the X-Y plane, then we know that



$I_{zz'} = I_{xx'} + I_{yy'}$

$\therefore \quad I_{zz'} = I_1 + I_2$ (i)

Also $I_{zz'} = I_3 + I_4$ (ii)

Adding Eqs. (i) and (ii), we get

$2I_{zz'} = I_1 + I_2 + I_3 + I_4$

But $I_1 = I_2$ and $I_3 = I_4$ (by symmetry)

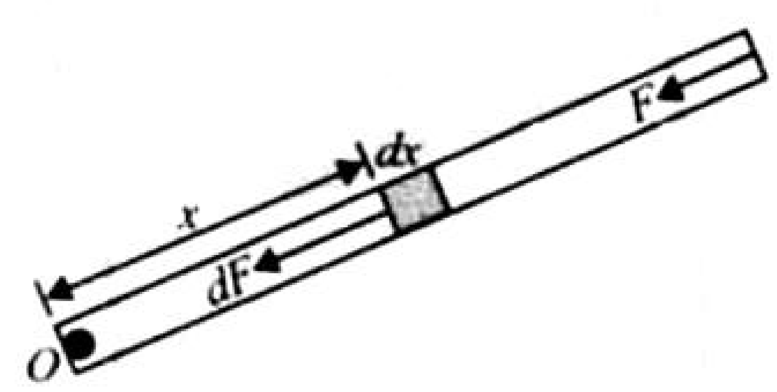
$\therefore \quad 2I_{zz'} = I_1 + I_1 + I_3 + I_3$

$= 2I_1 + 2I_3$

6. a. The force acting on the mass of liquid of length dx at a distance x from the axis of rotation O is as follows:

$dF = (dm) \times \omega^2$

$\therefore \quad dF = \frac{M}{L}dx \times x\omega^2$



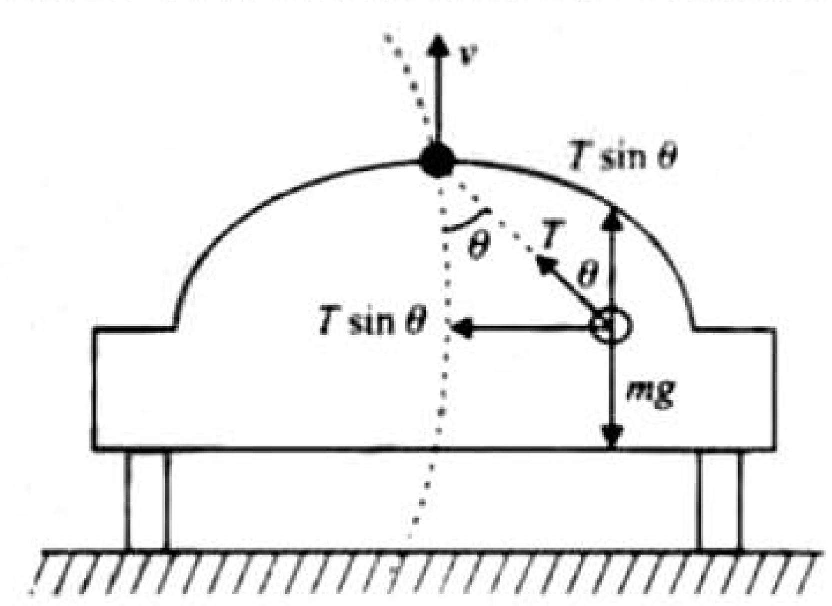
Therefore, the force acting at the other end is for the whole liquid in tube.

$$F\int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{M}{L}\omega^2\int_0^L x\,dx$$

$$= \frac{M}{L}\omega^2\left[\frac{x^2}{2}\right]^L = \frac{M}{L}\omega^2\left[\frac{L^2}{2} - 0\right]$$

$$= \frac{ML\omega^2}{2}$$

7. c. When the car is moving in a circular horizontal track of radius 10 m with a constant speed, then the bob is also undergoing a circular motion. The bob is under the influence of two forces.



i. T (tension in the rod)

ii. mg (weight of the bob)

Resolving tension, we get

$T\cos\theta = mg$ (i)

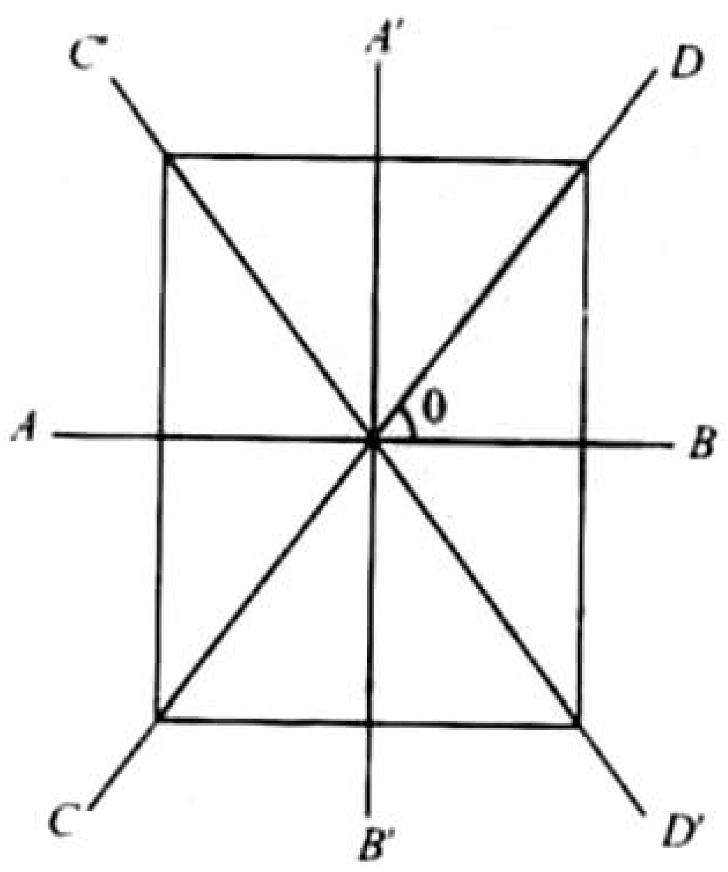
and $T\sin\theta = \frac{mv^2}{r}$ (ii)

(Here $T\sin\theta$ is producing the necessary centripetal force for the circular motion)

Dividing Eqs. (i) and (ii), we get

$$\tan\theta = \frac{v^2}{rg} = \frac{10\times10}{10\times10} = 1 \Rightarrow \theta = 45^\circ$$

8. a. $A'B' \perp AB$ and $C'D' \perp CD$



From symmetry, $I_{AB} = I_{A'B'}$ and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axis,

$$I_{ZZ} = I_{AB} I_{A'B'} = I_{CD} + I_{C'D'}$$

$$\Rightarrow \quad 2I_{AB} = 2I_{CD}$$

$$\therefore \quad I_{AB} = I_{CD}$$

Alternative method:

The relation between I_{AB} and I_{CD} should be true for all values of θ.

At $\theta = 0$, $I_{CD} = I_{AB}$

Similarly, at $\theta = \frac{\pi}{2}$

$I_{CD} = I_{AB}$ (By symmetry)

Keeping these things in mind, only option (a) is correct.

9. a., b., c. $\vec{\tau} = \frac{d\vec{L}}{dt}$

Given that $\vec{\tau} = \vec{A}\times\vec{L}$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$$

From the cross-product rule, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$. By the dot product definition, $\vec{L}\cdot\vec{L} = L^2$

Differentiating with respect to time, we get

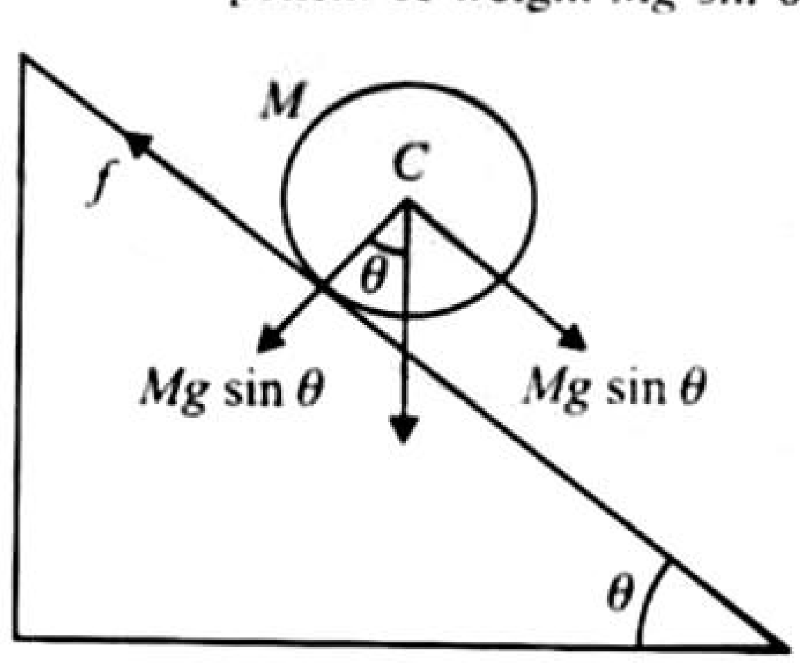
$$\vec{L}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

$$2\vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

Since $d\vec{L}/dt$ is perpendicular to $\vec{L}$

$$\Rightarrow \quad \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$$

10. c, d. As shown in the figure, the component of weight $Mg\sin\theta$ tends to slide the point of contact (of the cylinder with inclined plane) along its direction. The sliding friction acts in the opposite direction to this relative motion. Because of frictional force the cylinder rolls.



Thus frictional force aids rotation but hinders translational motion.

Applying $F_{net} = ma$ along the direction of inclined plane, we get,

$Mg\sin\theta - f = Ma_c$.

where a_c = acceleration of centre of mass of the cylinder

$\therefore \quad f = Mg\sin\theta - Ma_c$ (i)

where a_c = acceleration of centre of mass cylinder.

But $a_c = \frac{g\sin\theta}{1+\frac{I_c}{MR^2}} = \frac{g\sin\theta}{1+\frac{MR^2/2}{MR^2}}$

$= \frac{2}{3}g\sin\theta$ (ii)

From Eqs. (i) and (ii),

$$f = \frac{Mg\sin\theta}{3}$$

If θ is reduced, then frictional force is reduced.

11. a, d. Since no work is being done in the process, the sum of kinetic energy, translational kinetic energy, rotational kinetic energy and potential energy will be constant.

$$KE_A + mgh_A = KE_B = KE_C + mgh_C$$

$$KE_B > KE_A \text{ and } KE_B > KE_C$$

Now if $h_C > h_A$, then $KE_A < KE_C$.

12. b, c.

$$V_A = 0,\ V_B = \omega r,\ V_C = 2\omega r$$

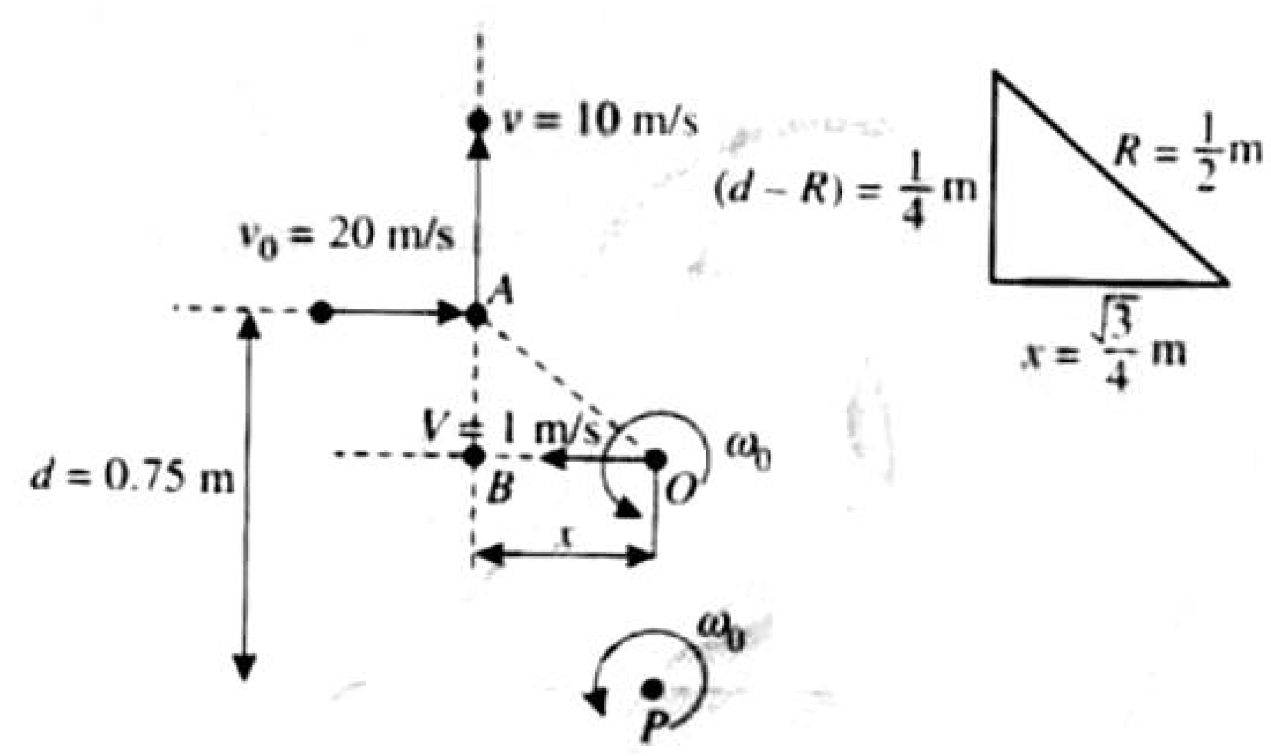
$\vec{V}_C - \vec{V}_A$ is towards the right and $\vec{V}_B - \vec{V}_C$ is towards left.

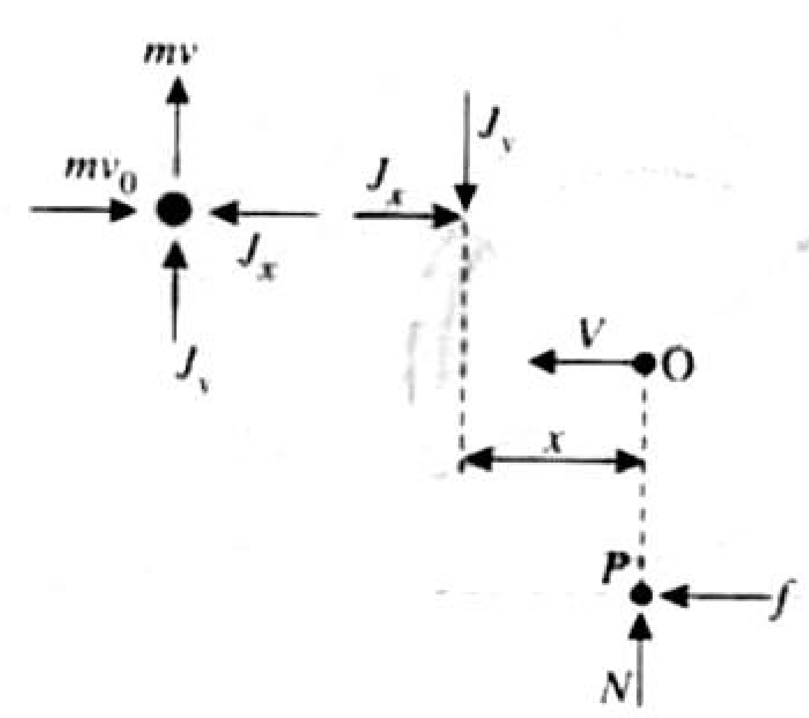
Hence, option (a) is incorret.

But in magnitude, option (c) is correct.

$$\vec{V}_C - \vec{V}_A = 2\omega r = 2\vec{V}_B$$

13. a, c. The friction and normal reaction on the ring is impulsive in nature.





Considering the motion of ball,

$$mv_0 - J_x = 0 \text{ or } J_x = mv_0$$

Hence $J_x = 0.1 \times 20 = 2$ kg m/s

and $J_y = mv = 0.1 \times 10 = 1$ kg m/s

Now considering the motion of ring.

$$J_x = MV_f - (-MV)$$

$$2 = MV_f + 2 \times 1 \Rightarrow V_f = 0$$

Hence the linear velocity of ring becomes zero just after collision.

Conserving angular momentum of system (ring + ball) about point of contact (P).

$$I_P\omega_0(\hat{k}) + mv_0 d(-\hat{k}) = I_0\vec{\omega} + mvx(-\hat{k})$$

$$I_0\vec{\omega} = I_P\omega_0\hat{k} + m[vx - v_0 d]\hat{k}$$

As ring is rolling $\omega_0 = \frac{V}{R} = \frac{1}{1/2} = 2$ rad/s

$$2 \times \left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2} \times 2 \times \left(\frac{1}{2}\right)^2 \times 2\hat{k} + 0.1\left[10 \times \frac{\sqrt{3}}{4} - 20 \times \frac{1}{4}\right]\hat{k}$$

$$2 \times \left(\frac{1}{2}\right)^2 \vec{\omega} = \frac{3}{2}\hat{k} + 0.1 \times \frac{10}{4}[3-2]\hat{k}$$

Hence $\omega \neq 0$ and will be in counter clockwise direction. Now friction will be kinetic in nature and will act in leftward direction.

14. b, d.

$$V_{es} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho \cdot \frac{4}{3}\pi R^3}{R}} = \sqrt{\frac{8\pi G\rho}{3}}R$$

$$V_{es} \propto R$$

Surface area of $P = A = 4\pi R_P^2$

Surface area of $Q = 4A = 4\pi R_Q^2$

$\Rightarrow \quad R_Q = 2R_P$

Mass R is $M_R = M_P + M_Q$

$$\rho\frac{4}{3}\pi R_R^3 = \rho\frac{4}{3}\pi R_P^3 + \rho\frac{4}{3}\pi R_Q^3$$

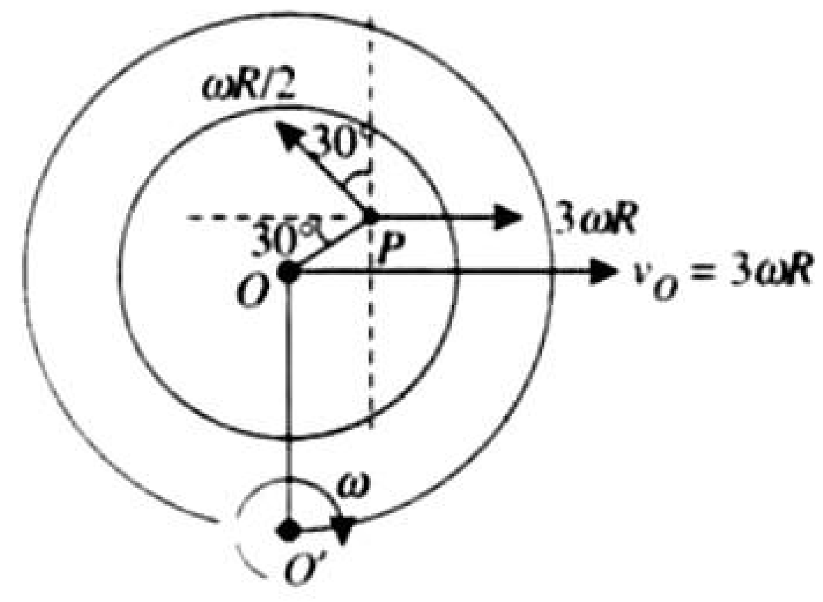
$$\Rightarrow \quad R_R^3 = R_P^3 + R_Q^3 = 9R_P^3$$

$$R_R = 9^{1/3}R_P \Rightarrow R_R > R_Q > R_P$$

Therefore $V_R > V_Q > V_P$

$$\frac{V_R}{V_P} = 9^{1/3} \text{ and } \frac{V_P}{V_Q} = \frac{1}{2}$$

15. As ring is rolling the point of contact of ring with ground (O') will be instantaneous center of rotation. Hence for pure rolling $= \omega(3R) = 3R\omega$ and its direction is along positive x.



Instantaneous center of rotation

The velocity at the point P

$$v_P = 3R\omega\hat{i} - \frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 30°\hat{k}$$

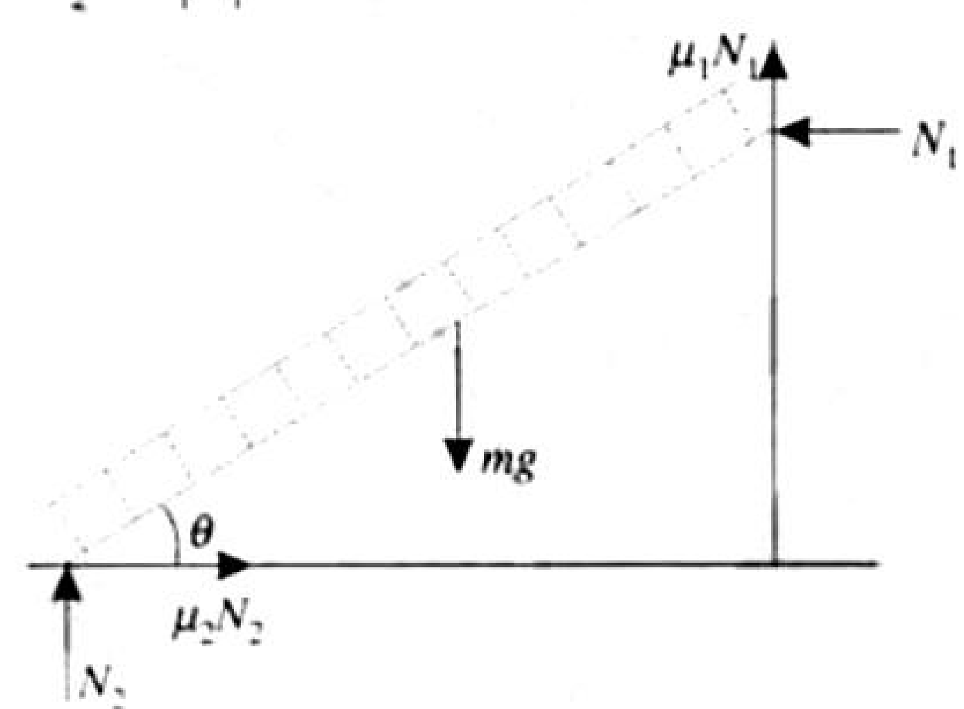
$$v_P = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

16. c, d.

Consider of translational equilibrium

$$N_1 = \mu_2 N_2 \quad \text{(i)}$$

$$N_2 + \mu_1 N_1 = Mg \quad \text{(ii)}$$



Solving $N N_2 = \frac{mg}{1 + \mu_1\mu_2}$

$$N_1 = \frac{\mu_2 mg}{1 + \mu_1\mu_2}$$

Applying torque equation about corner (left) point on the floor

$$mg\frac{\ell}{2}\cos\theta = N_1\ell\sin\theta + \mu_1 N_1\ell\cos\theta$$

Solving $\tan\theta = \frac{1 - \mu_1\mu_2}{2\mu_2}$

Linked Comprehension Type

1. c. $\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2$ and $\frac{1}{2}2I(\omega)^2 = \frac{1}{2}kx_2^2$

Dividing them, we get

$$\frac{x_1}{x_2} = \sqrt{2}$$

2. a. Let final angular velocity be ω_1. Applying conservation of angular momentum:

$$(I + 2I)\omega_1 = I(2\omega) + 2I\omega$$

$$\Rightarrow \quad \omega_1 = \frac{4\omega}{3}$$

Now angular impulse = change in angular momentum:

$$\tau t = 2I(\omega_1 - \omega)$$

$$\Rightarrow \quad \tau t = 2I\frac{\omega}{3} \quad \Rightarrow \quad t = \frac{2I\omega}{3\tau}$$

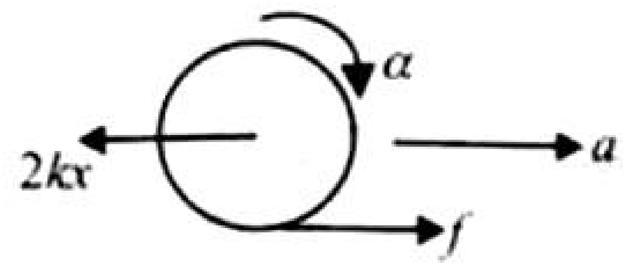
3. b. $KE_i - KE_f$

$$\Rightarrow \quad \frac{1}{2}I(2\omega)^2 + \frac{1}{2}2I\omega^2 - \frac{1}{2}3I\omega^2 = \frac{1}{3}I\omega^2$$

4. d. $f - 2kx = Ma, fR = -I\alpha$

$$\Rightarrow \quad fR = -(1/2)MR^2(a/R)$$

Solve to get net force $= Ma = -\frac{4kx}{3}$



Negative sign is because this force will act in the backward direction.

5. d. From the previous question, $a = -\frac{4kx}{3M}$

Comparing with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{4k}{3M}}$$

6. c. From the previous question, we can get

$$f = \frac{2kx}{3}$$

$$f_{max} = \mu Mg$$

For disc not to slip, $f_{max} \geq f \Rightarrow x \leq \frac{3}{2}\frac{\mu Mg}{k}$ (i)

From conservation of energy:

$$\frac{1}{2}(2k)x^2 = \frac{1}{2}I_{CM}\,\omega^2 + \frac{1}{2}MV_0^2$$

$$\Rightarrow \quad x = \sqrt{\frac{3}{4}\frac{MV_0^2}{k}}$$

Putting the value of x in Eq. (i), we get

$$\Rightarrow \quad V_0 \leq \mu g\sqrt{\frac{3M}{k}}$$

7. d. Angular speed about the instantaneous axis passing through centre of mass is ω for both the cases.

8. a. Instantaneous axis passing through cm is vertical in both the cases.

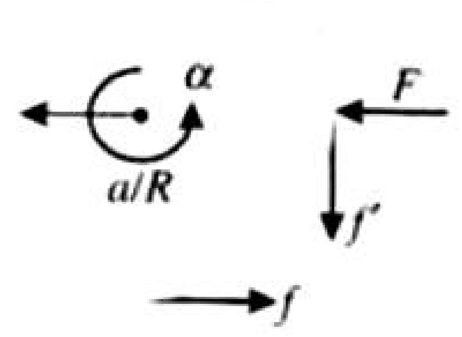
Integer Answer Type

1. (4) There are two possibilities for applied force by string.

(i) If net force applied by the rod is considered to be 2 N.

(ii) If only normal reaction applied by the rod is considered to be 2 N.

Let us consider 1st possibility that If net force applied by the rod is considered to be 2 N.



Hence net force applied by the rod

$$\sqrt{f'^2 + F^2} = 2 \quad \text{(i)}$$

Applying torque equation about point of contact

$$FR - f'R = 2mR^2\frac{a}{R}$$

$$F - f' = 2ma = 1.2 \quad \text{(ii)}$$

From (i) and (ii), $(1.2 + f')^2 + f'^2 = 2^2$

$$2f'^2 + 2.4f' + 1.44 = 4$$

$$f'^2 + 1.2f' + 0.72 - 2 = 0$$

$$f'^2 + 1.2f' - 1.28 = 0$$

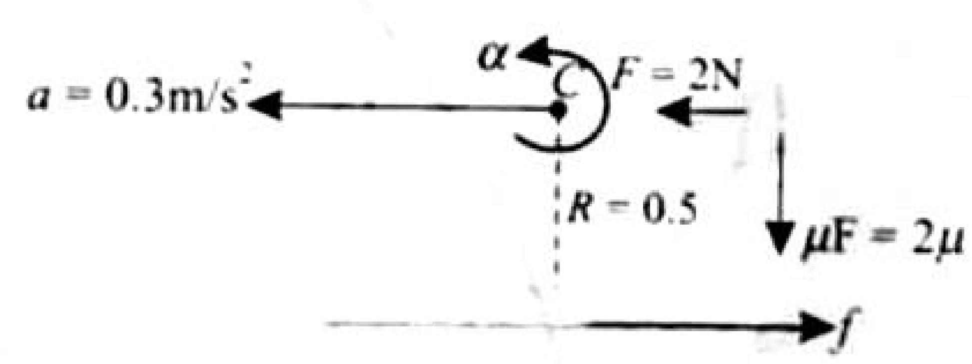
$$f' = \frac{-1.2 \pm \sqrt{1.44 + 4 \times (1.28)}}{2}$$

$$= 0.6 \pm \sqrt{0.36 + 1.28} = -0.6 \pm \sqrt{0.64} = 0.68$$

From equation (ii), $F = 1.88$

$$\mu = \frac{0.68}{1.88} = \frac{P}{10} \quad \Rightarrow \quad P = 3.16 \approx 4$$

Now let us consider the possibility that if only normal reaction applied by the rod is considered to be 2 N.



From F.B.D of ring, writing equation of motion of ring

$$2 - f = 2[0.3] \quad \Rightarrow \quad f = 2 - 0.6 = 1.4 \text{ N} \quad \text{(i)}$$

As disc is rolling, $A = R\alpha \quad \Rightarrow \quad 0.3 = \alpha[0.5]$

$$\alpha = \frac{3}{5} \text{ rad/s} \quad \text{(ii)}$$

Now torque equation, $\tau_c = I_c\alpha$

$$fR - 2\mu R = mR^2\alpha$$

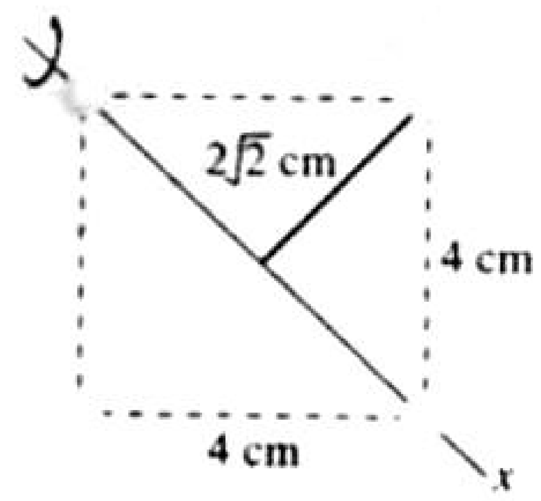
$$f - 2\mu = mR\alpha$$

$$1.4 - 2\mu = \frac{2}{2}\left(\frac{3}{2}\right)$$

$$0.8 = 2\mu \quad \Rightarrow \mu = 0.4 = \frac{P}{10} \quad \because \quad P = 4$$

2. (9) The moment of inertia of the system about the diagonal of the square

$$I = \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2 + Mx^2\right)2$$



$$= \left(\frac{2}{5}MR^2\right)2 + \left(\frac{2}{5}MR^2\right)2 = (Mx^2)2$$

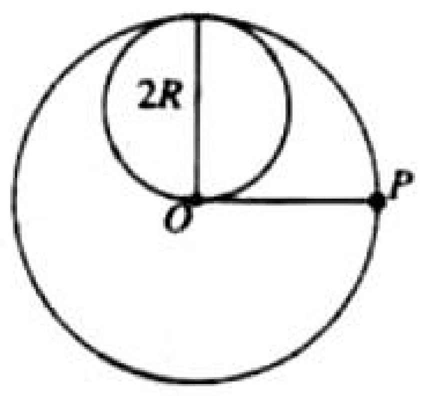
$$= 4\left(\frac{2}{5}MR^2\right) + 2Mx^2 = \frac{8}{5}MR^2 + 2mx^2$$

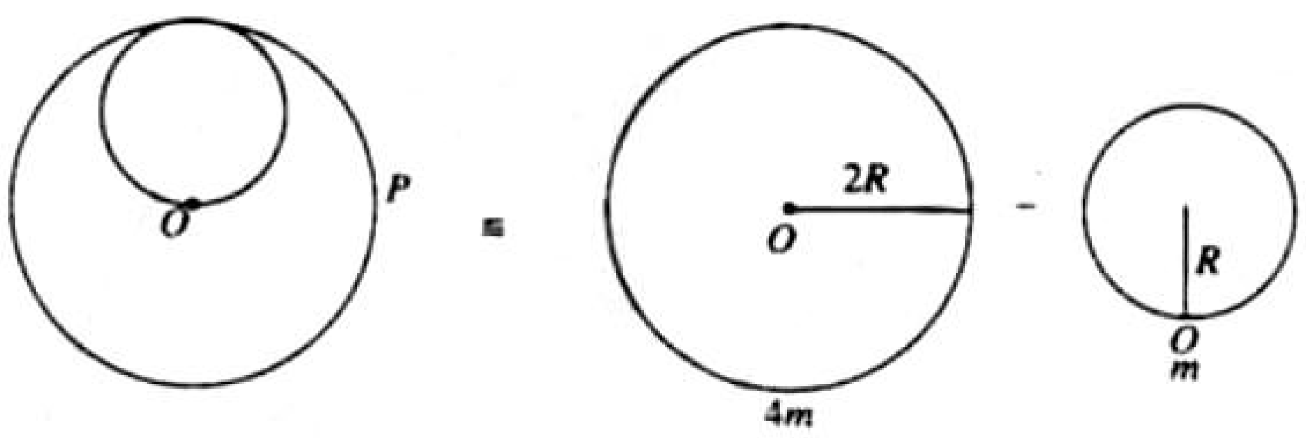
$$= \left[\frac{8}{5}\times 0.5\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\times(0.5)\times(4\times 2)\right]10^{-4}$$

$$= \left[\frac{5}{5}+8\right]\times 10^{-4} = 9\times 10^{-4} = N\times 10^{-4}$$

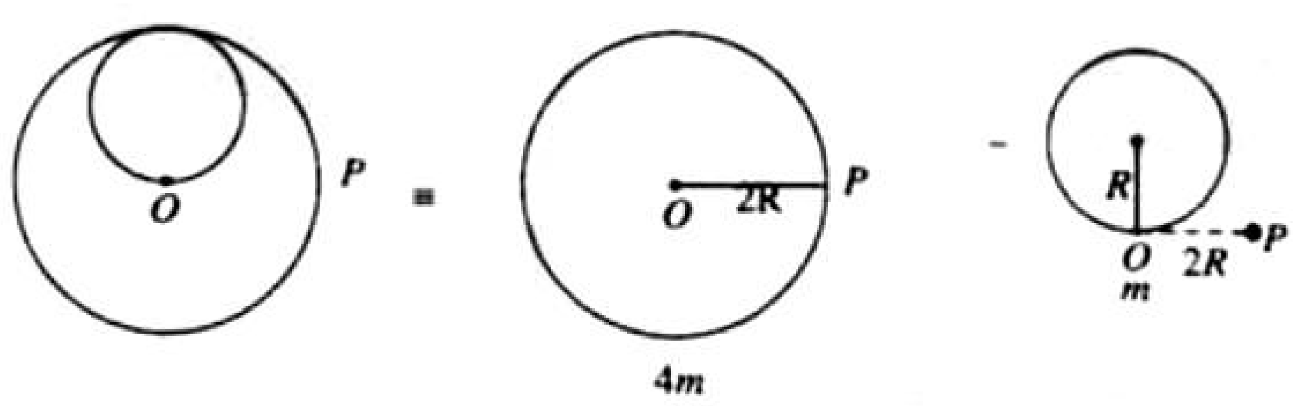
So, $N = 9$

3. (3)





$$I_O = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2 = mR^2\left[8 - \frac{3}{2}\right] = \frac{13}{2}mR^2$$

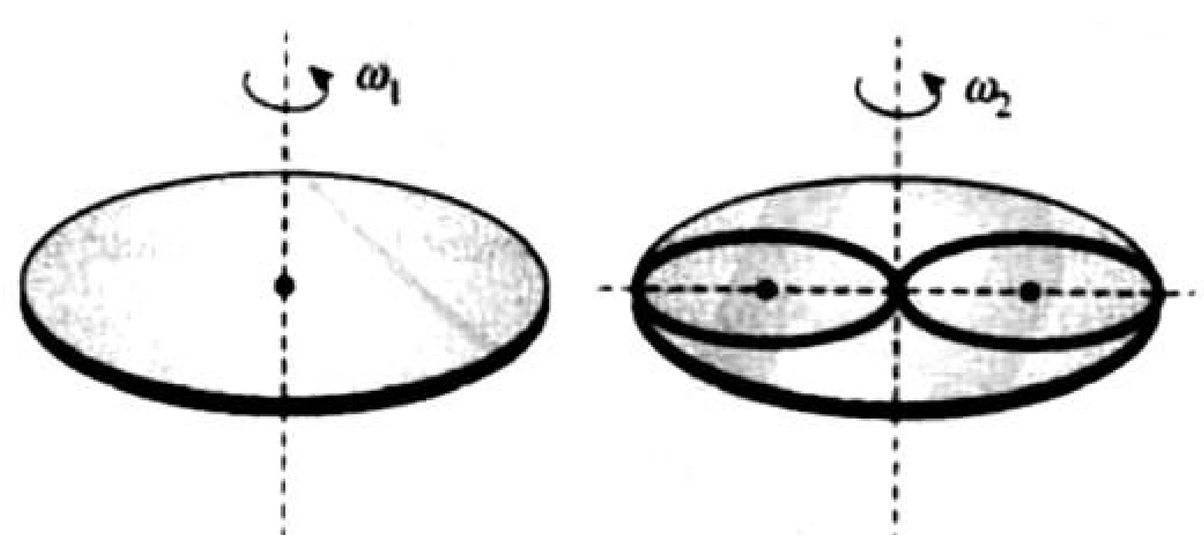


$$I_P = \frac{3}{2}(4m)(2R)^2 - \left[\frac{mR^2}{2} + m\{(2R)^2 + R^2\}\right]$$

$$= 24mR^2 - \frac{11}{2}mR^2 = \frac{37}{2}mR^2$$

$$\frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

4. (8)



Only disc is rotating   Rotation of disc with ring

Moment of inertia of the disc about its axis

$$I_1 = \frac{1}{2}MR^2 = \frac{1}{2}\times 50\text{ kg}\times(0.4\text{ m})^2 = 4\text{ kg m}^2$$

$\therefore$ Initial angular momentum of the disc is $L_1 = I_1\omega_1$

When two uniform circular rings are placed symmetrically on the disc such that they are touching each other along the axis of the disc and are horizontal as shown in figure (b). Then the moment of inertia of the system about the given axis

$$I_2 = \frac{1}{2}MR^2 + 2mr^2 + 2mr^2 = \frac{1}{2}MR^2 + 4mr^2$$

$$= \frac{1}{2}\times 50\text{ kg}\times(0.4\text{ m})^2 + 4\times 6.25\text{ kg}\times(0.2\text{ m})^2$$

$$= 4\text{ kg m}^2 + 1\text{ kg m}^2 = 5\text{ kg m}^2$$

Let ω_2 be the final angular speed of the system.

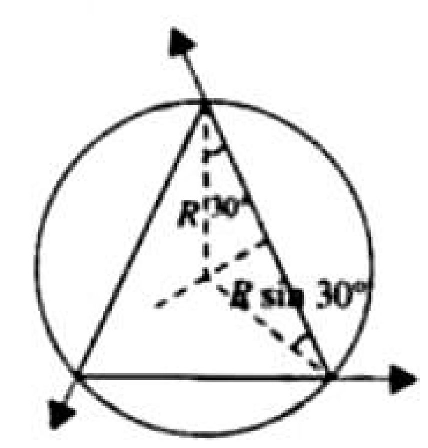
Final angular momentum of the system is $L_2 = I_2\omega_2$

According to law of conservation of angular momentum, we get

$$L_1 = L_2 \text{ or } I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{4\text{ kg m}^2}{5\text{ kg m}^2}\times 10\text{ rad s}^{-1} = 8\text{ rad s}^{-1}$$

5. (2) $\tau_{net} = 3\tau_1 = 3Fr\sin 30 = 3(0.5)(0.5)\frac{1}{2} = \frac{3}{8}$ N-m



$$I = \frac{1.5(0.5)^2}{2} = \frac{3}{16}$$

$$\alpha = \frac{\tau}{I} = 2\text{ rad/s}^2$$

$$\omega = \alpha t = 2\text{ rad/s}$$

6. (4) Since net torque about centre of rotation is zero, so we can apply conservation of angular momentum of the system about center of disc

$$L_i = L_f$$

$0 = I\omega + 2mv\,(r/2)$; comparing magnitude

$$\therefore \quad \left(\frac{0.45\times 0.5\times 0.5}{2}\right)\omega = 0.05\times 9\times\frac{0.5}{2}\times 2$$

$$\therefore \quad \omega = 4$$

7. (7) Kinetic energy of a purely rolling disc

$$= \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{m}{2}\right)V^2 = \frac{3}{4}mV^2$$

Using conservation of energy on both discs and equating their final energies

$$\frac{3}{4}mV_1^2 + mg\cdot 30 = \frac{3}{4}mV_2^2 + mg\cdot 27$$

$$3mg = \frac{3}{4}m(V_2^2 - V_1^2)$$

$$4g = V_2^2 - 9$$

$$V_2^2 = 49$$

$$V_2 = 7$$

8. (6) Moment of inertia of a hollow sphere is $I_0 = \frac{2}{3}mR^2$

Here, for sphere A:

$$I_A = \int_0^R \frac{2}{3} r^2 \cdot 4\pi r^2\, dr K \frac{r}{R}$$

$$= \frac{8\pi K}{3R}\int_0^R r^5 dr = \frac{8\pi K}{3R}\,\frac{R^6}{6}$$

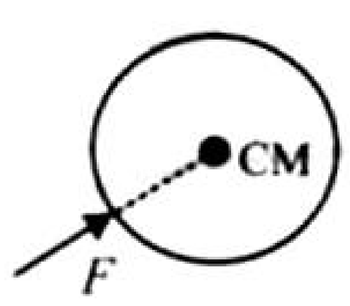
Also, for sphere B:

$$I_B = \int_0^R \frac{2}{3} r^2 \cdot 4\pi r^2 dr \frac{K}{R^5}\cdot r^5 = \frac{8\pi K}{3R^5}\cdot\frac{R^{10}}{10}$$

Ratio, $\frac{I_B}{I_A} = \frac{6}{10}$

Assertion-Reasoning Type

1. d. **For statement 1:** Force may be acting through centre of mass as shown in the figure. For this, torque about CM will be zero. The force F will produce an acceleration in the CM.

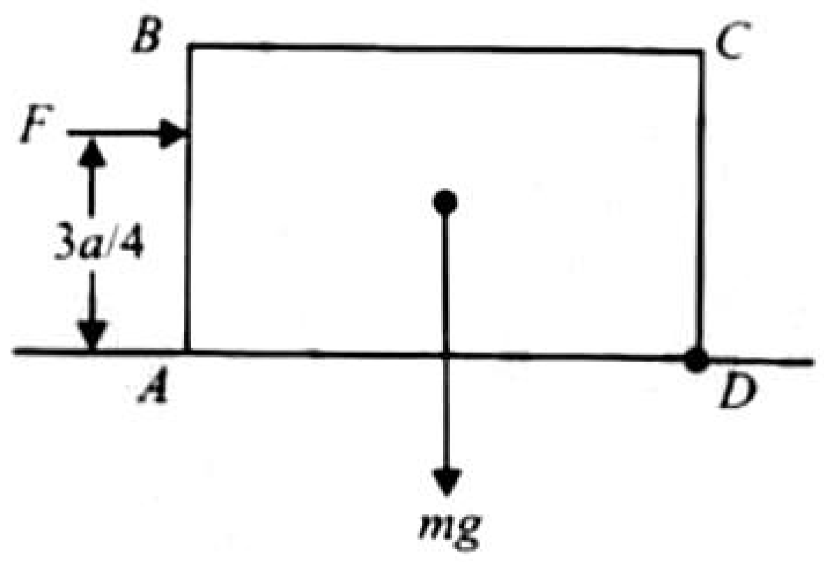


2. d. $a = \frac{g\sin\theta}{1 + \frac{I_{CM}}{mR^2}}$, I_{CM} of hollow cylinder is less, so it will have more acceleration and will take less time to reach the bottom.

Fill in the Blanks Type

1. Taking moment of force about D,

$$F = \frac{3a}{4} = mg \times \frac{a}{2} = \frac{2}{3}mg$$



2. Since no force and hence no torque is applied, the angular momentum remains constant

$\therefore \quad I_1\omega_1 = I_2\omega_2$

$$\therefore \quad \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{\frac{ML^2}{12}\times\omega_0}{\frac{ML^2}{12} + 2m\times\left(\frac{L}{2}\right)^2} = \frac{M\omega_0}{M + 6m}$$

3. $x = A\cos\omega t$

$$\Rightarrow \quad \frac{dx}{dt} = -A\omega\sin\omega t$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -A\omega^2\cos\omega t$$

$\therefore \quad$ |Max acceleration| $= A\omega^2$

$$\therefore \quad \alpha_{max} = \frac{A\omega^2}{R}$$

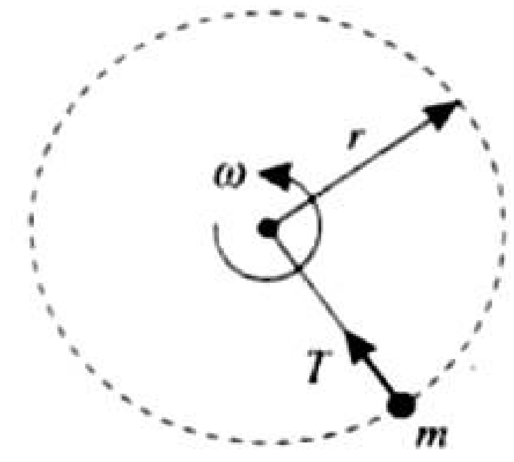
Max torque $= I\alpha_{max}$

$$= \frac{1}{2}MR^2 \times \frac{A\omega^2}{R} = \frac{1}{2}MRA\omega^2$$

4. Let at any instant of time t, the radius of the horizontal surface be r.

$$T = mr\omega^2 \qquad \text{(i)}$$

where m is the mass of the stone and ω is the angular velocity at that instant of time t.



At time t

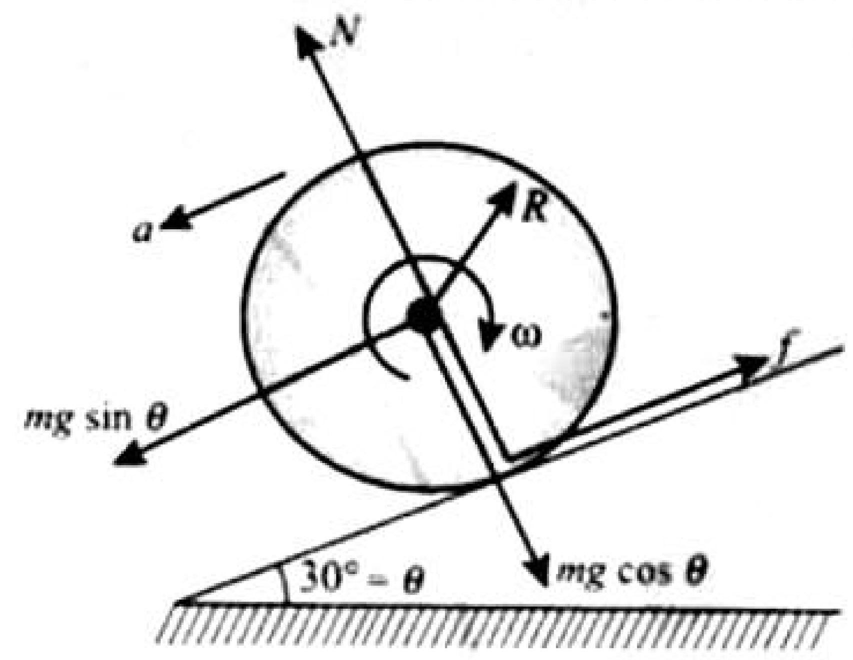
Also, $L = I\omega \qquad$ (ii)

From Eqs. (i) and (ii)

$$T = \frac{mrL^2}{I^2} = \frac{mL^2}{(mr^2)^2}\times r$$

$\Rightarrow \quad T \propto r^{-3}$ ($\because$ m, L are constant)

5. Given that the body is rolling up the inclined plane. Therefore, the velocity of the centre of mass is up the inclined plane. The component of weight ($mg\sin\theta$) is trying to move the point of contact downwards. Therefore, frictional force will act upwards.



From force diagram,

$$mg\sin\theta = f\, ma_c \qquad \text{(i)}$$

For rotational motion about the centre of mass of disc O.

$$f\times R = I\alpha$$

$\Rightarrow \quad$ But $a_c = \alpha R$

$$\Rightarrow \quad \alpha = \frac{a_c}{R} f\times R = \frac{Ia_c}{R}$$

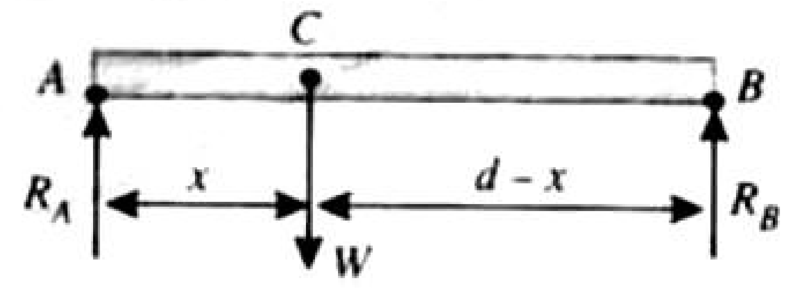
$$\Rightarrow \quad a_0 = \frac{fR^2}{I} = \frac{fR^2}{\frac{1}{2}mR^2} = \frac{2f}{m} \qquad \text{(ii)}$$

From Eqs. (i) and (ii),

$\Rightarrow \quad 3f = mg\sin\theta = mg\sin 30°$

$$\Rightarrow \quad f = \frac{mg}{6}$$

6. Let R_A and R_B be the reactions at the knife edges A and B, respectively (as shown in figure). For the rod to be in equilibrium in a horizontal position, the moment of forces about the knife edges must be equal to zero.



Moment of forces about A,

$$R_A\times 0 + xW - R_Bd = 0$$

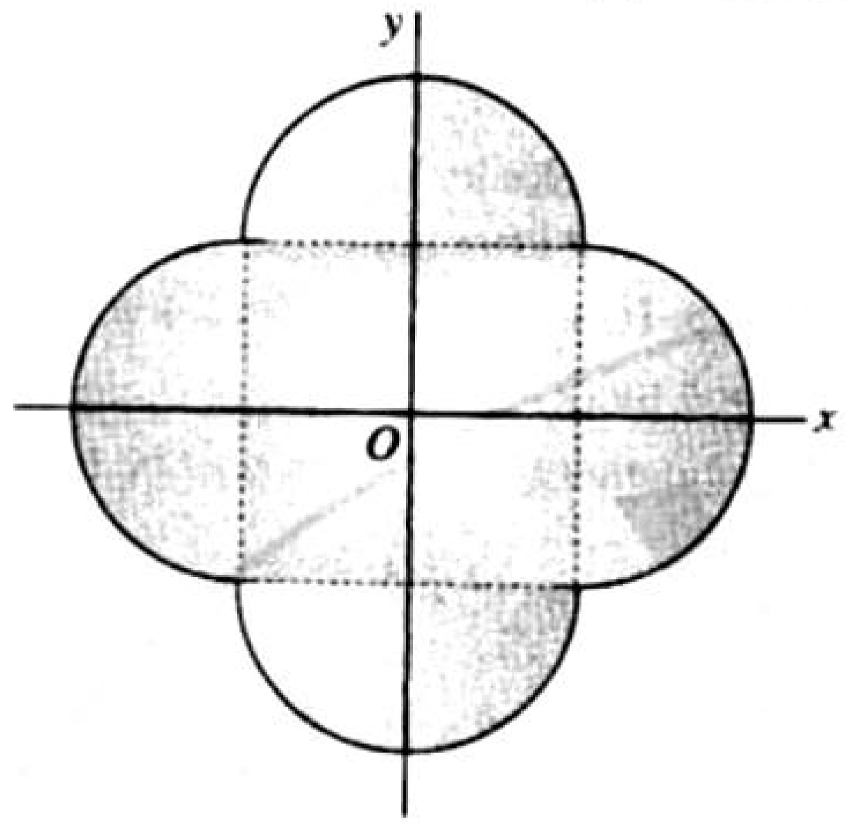
i.e., $R_B = \frac{xW}{d}$

Moment of forces about B,

$$R_A d - (d - x) W - R_B 0 = 0$$

i.e., $R_A = \left(\frac{d - x}{d}\right) W$

7. Assuming symmetric lamina to be in xy plane, we will have



$I_x = I_y$ (Since the mass distribution is the same about the x-axis and y-axis)

$$I_x = I_y = I_z \text{ (perpendicular-axis theorem)}$$

It is given that

$$I_z = 1.6\, Ma^2$$

Hence, $I_x = I_y = \frac{I_z}{2} = 0.8 Ma^2$

Now, according to the parallel-axis theorem, we get

$$I_{AB} = I_{XT} M(2a)^2$$
$$= 0.8Ma^2 + 4Ma^2 = 4.8Ma^2$$

True/False Type

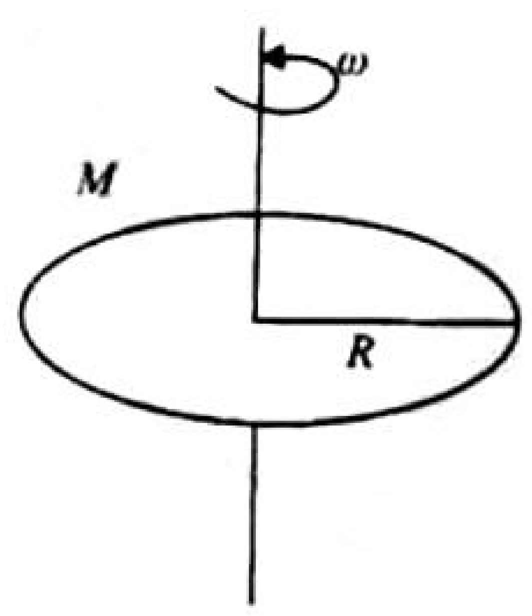
1. **False:** $\tau = I_\alpha$, therefore $\alpha = \frac{\tau}{L}$

τ = Force × Perpendicular distance

Torque is the same in both the cases. But since I will be different due to different mass distribution about the axis, therefore α will be different.

2. **False:** $\vec{\tau} = \frac{\vec{dt}}{dt}$

Since $\vec{\tau} = 0$, $\vec{L}$ = const



$\therefore \quad I_1\omega_1 = I_2\omega_2$

$$I_1 = \frac{1}{2} MR^2, \omega_1 = \omega$$

$$I_2 = \frac{1}{2}MR^2 + \frac{1}{2}\frac{M}{4}R^2$$

$$= \left(\frac{4+1}{8}\right)MR^2 = \frac{5}{8}MR^2$$

ω_2 = ?

$$\omega_2 = \frac{I_1\omega_1}{I_2} \frac{\frac{1}{2}MR^2 \times \omega}{\frac{5}{8}MR^2} = \frac{8}{2 \times 5}\omega = \frac{4}{5}\omega$$

3. **False:** Total energy of the ring = $(KE)_{Rotation} + (KE)_{Translational}$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2}mV_c^2$$

$$= \frac{1}{2} \times mr^2\omega^2 + \frac{1}{2}m(r\omega)^2$$

$\therefore \quad I = mr^2, V_c = r\omega$

Total KE of the cylinder

$$= (KE)_{Rotation} + (KE)_{Translational}$$

$$= \frac{1}{2} = I'\omega'^2 + \frac{1}{2}MV_c'^2$$

$$= \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega'^2 + \frac{1}{2}M(r\omega')^2$$

$$= \frac{3}{4}Mr^2\omega'^2 \qquad \text{(i)}$$

Equating Eqs. (i) and (ii), $mr^2\omega^2 = \frac{3}{4}Mr^2\omega'^2$

$\Rightarrow \quad \frac{\omega'^2}{\omega^2} = \frac{4m}{3M} = \frac{4}{3} \times \frac{0.3}{0.4} = 1$

$\Rightarrow \quad \omega' = \omega$

Both will reach at the same time.

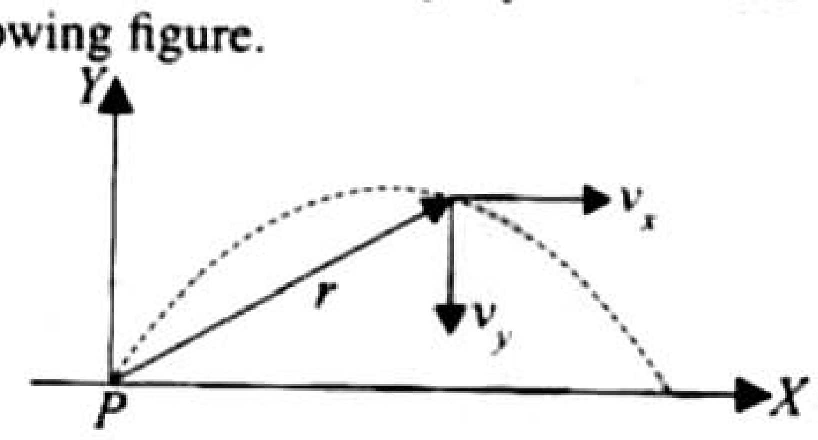
4. **False:** Since no external force is acting on the two particle system,

$\therefore \quad a_{CM} = 0$

$\Rightarrow \quad V_{CM}$ = constant

Subjective Type

1. Let us take the origin at P, x-axis along the horizontal and y-axis along the vertically upward direction as shown in the following figure.



For horizontal motion during the time 0 to t,

$$v_x = v_0 \cos 45° = \frac{v_0}{\sqrt{2}}$$

and $\quad x = v_x t = \frac{v_0}{\sqrt{2}} \frac{v_0}{g} = \frac{v_0^2}{\sqrt{2}g}$

For vertical motion, using $v_y = v_{oy} - gt$

$$v_y = v_0 \sin 45° - g\left(\frac{v_0}{g}\right) = \frac{(1-\sqrt{2})}{\sqrt{2}} v_0$$

and $y = (v_0 \sin 45°)\, t - \frac{1}{2} g t^2$

$$\Rightarrow \quad y = \frac{v_0}{\sqrt{2}}\left(\frac{v_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{2g}(\sqrt{2} - 1)$$

The angular momentum of the particle at time t about the origin is

$$L = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$= (\vec{i}x + \vec{j}y) \times (v_x \vec{i} + v_y \vec{j})$$

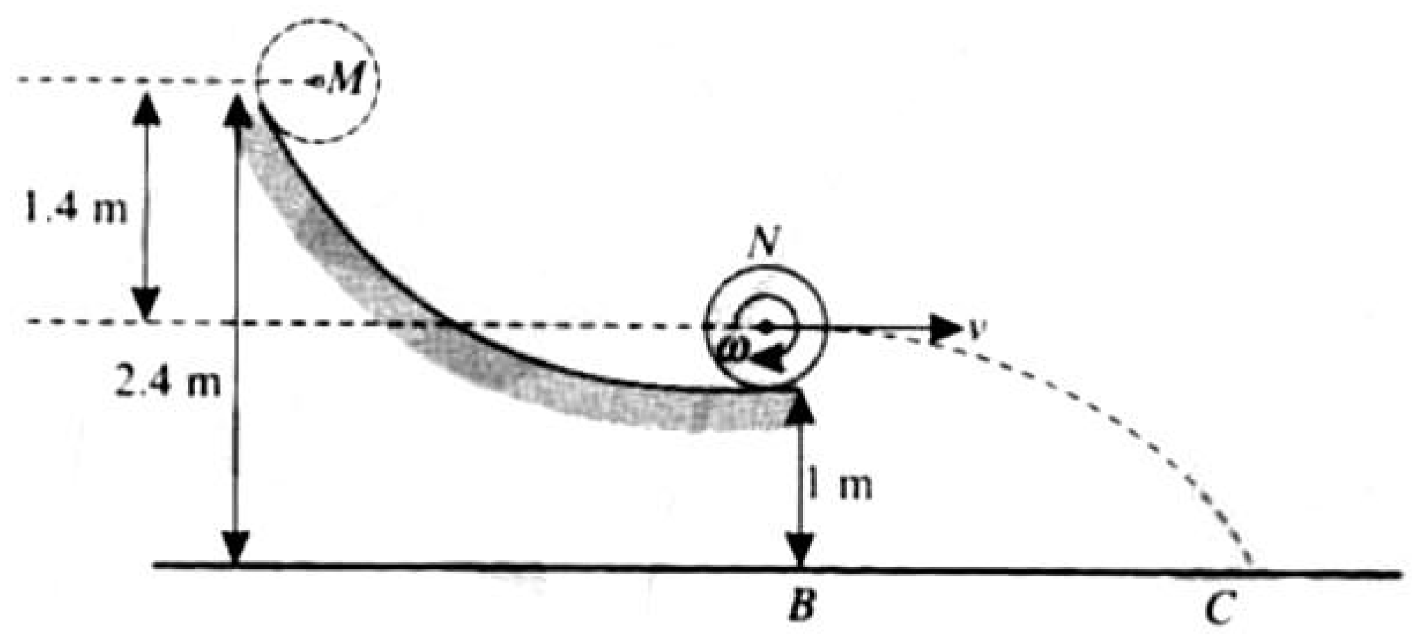
$$= m\,(kxv_y - kyv_x)$$

$$= m\,\vec{k}\left[\left(\frac{v_0^2}{\sqrt{2}g}\right)\frac{v_0}{\sqrt{2}}(1 - \sqrt{2}) - \frac{v_0^2}{2g}(\sqrt{2} - 1)\frac{v_0}{\sqrt{2}}\right]$$

$$= -\vec{k}\,\frac{mv_0^3}{2\sqrt{2}g}$$

Thus, the angular momentum of the particle is $mv_0^3/(2\sqrt{2}g)$ in the negative Z-direction, i.e., perpendicular to the plane of motion, going into the plane.

2. Applying law of conservation of mechanical energy at point M and point N



$$\Delta K + \Delta U = 0$$

Increase in KE = Decrease in PE

$$\left(\frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2 - 0\right) = (mg(2.4) - mg(1))$$

Since the sphere is of rolling without slipping $\upsilon = r\omega$ hence $\omega = \frac{\upsilon}{r}$ where r is the radius of the sphere

Also $I = \frac{2}{5}mr^2$

$$\therefore \quad \frac{1}{2}m\upsilon^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{\upsilon^2}{r^2} = mg(2.4) - mg(1)$$

$$\Rightarrow \quad \upsilon = 4.43 \text{ m/s}$$

After point N, the sphere takes a parabolic path. Considering the motion of sphere in vertical direction

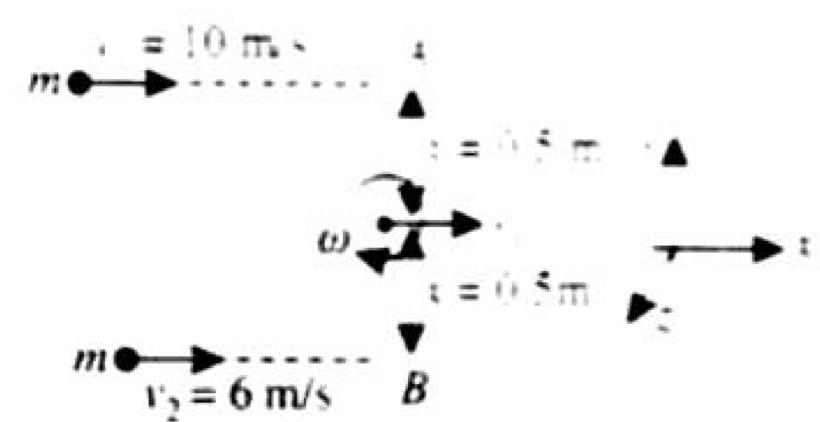
$$u_y = 0,\ s_y = 1 \text{ m},\ a_y = 9.8 \text{ m/s}^2$$

Using $s_y = u_y t + \frac{1}{2}a_y t^2$

$$1 = 4.9t^2 \quad \Rightarrow \quad t = \frac{1}{\sqrt{4.9}} = 0.45 \text{ sec}$$

In horizontal direction, the velocity of sphere remains constant. Hence $x = v \times t = 4.43 \times 0.45 = 2$ m

3. Considering bar + particle as system and applying conservation of linear momentum just before and just after collision



$$m_1 \times v_1 + m_2 \times v_2 = (M + m_1 + m_2)v_C$$

where v_C is the velocity of center of mass of the bar and particles sticked on it after collision

$$0.08 \times 10 + 0.08 \times 6 = (0.16 + 0.08 + 0.08)v_C$$

$$\Rightarrow \quad v_C = 4 \text{ m/s}$$

Initial kinetic energy of system will be only translator nature.

$$K_{\text{initial}} = \frac{1}{2}m_1\upsilon_1^2 + \frac{1}{2}m_2\upsilon_2^2$$

$$= \frac{1}{2}0.08 \times 10^2 + \frac{1}{2} \times 0.08 \times 6^2 = 5.44 \text{ J} \qquad \text{(i)}$$

∴ Translational kinetic energy after collision

$$K_{\text{translatory}} = \frac{1}{2}(M + m_1 + m_2)v_C^2 = 2.56 \text{ J} \qquad \text{(ii)}$$

Now applying conservation of angular momentum of the bar and two particle system about the center of the bar. Since external torque is zero, the initial angular momentum should be equal to final angular momentum.

Initial angular momentum $L_{\text{initial}} = m_1\upsilon_1 x - m_2\upsilon_2 x$

$$= 0.08 \times 10 \times 0.5 - 0.08 \times 6 \times 0.5$$

$$= 0.4 - 0.24 = 0.16 \text{ kg m}^2\text{s}^{-1} \text{ (in clockwise direction)}$$

Final angular momentum $L_{\text{final}} = I\omega$

$$L_{\text{final}} = \left[\frac{Ml^2}{12} + m_1x^2 + m_2x^2\right]\omega$$

$$= \left[\frac{(0.16)(\sqrt{3})^2}{12} + 2 \times 0.08 \times (0.5)^2\right]\omega = 0.08\omega$$

$$\therefore \quad 0.08\omega = 0.16 \quad \Rightarrow \quad \omega = 2 \text{ rad/s} \qquad \text{(iii)}$$

The rotational kinetic energy

$$K_{\text{rotation}} = \frac{1}{2}I\omega^2 = \frac{1}{2} = 0.08 \times 2^2 = 0.16 \text{ J} \qquad \text{(iii)}$$

The final kinetic energy

$$K_{\text{final}} = K_{\text{translational}} + K_{\text{rotational}}$$

$$= 2.56 + 0.16 = 2.72\text{J} \qquad \text{(iv)}$$

The change is K.E. = Initial K.E. – Final K.E.

$$= 5.44 - 2.72 = 2.72 \text{ J}$$

4. The below figure shows the forces acting on the carpet in two cases:

   i. when the radius of the roll is R

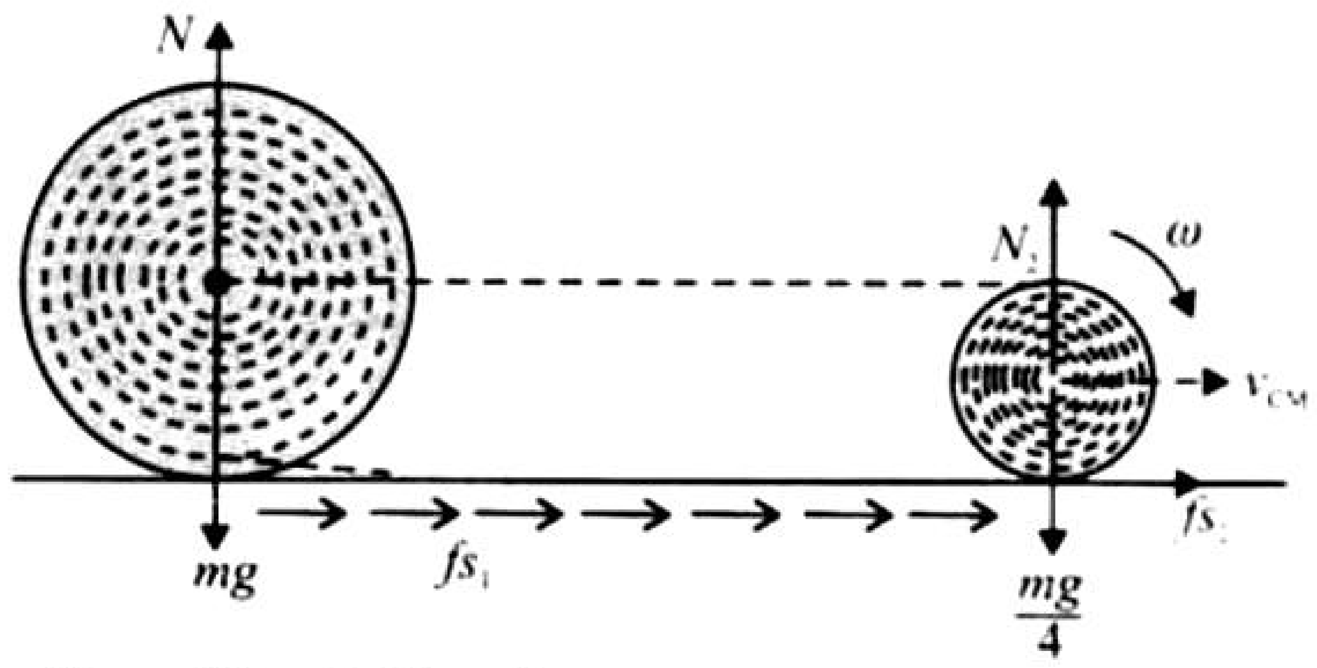
   ii. when the radius is $R/2$

Mass of the roll of radius $R/2 = M/4$

(Mass is proportional to the area of cross section)

The mass of the part of the carpet spread on the floor = (3/4) M.

There has to be a frictional force that is too static in nature on the carpet once it starts unrolling. Otherwise, why should the centre of mass of the system (carpet of mass m) move towards the right? Is such an unrolling possible on a smooth surface? Does the centre of mass of the system (M) move in the vertical direction? If yes, why?



Here, $W_{f_s} = 0$, $W_N = 0$

Therefore, $W_{ncf} = 0$.

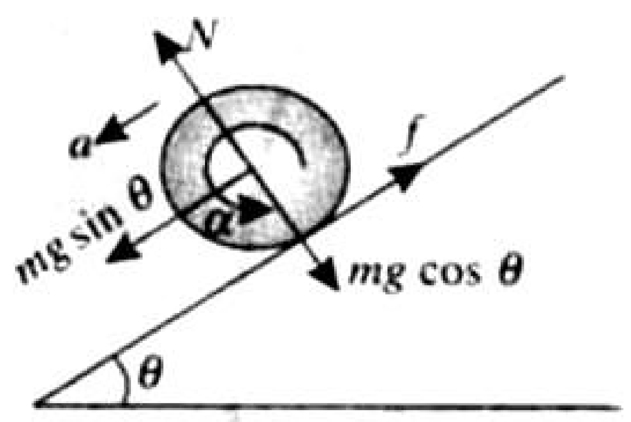
The mechanical energy of the system will remain constant. The part of the carpet spread on the ground will have neither kinetic energy nor potential energy.

Therefore, $MgR = \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) + \frac{1}{2}\left(\frac{M}{4}\right)v_{CM}^2 + \frac{1}{2}\left(\frac{1}{2}\frac{M}{4}\left(\frac{R}{2}\right)^2\right)\omega^2$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{M}{8}v_{CM}^2 + \frac{1}{16}M\left(\frac{R}{2}\right)^2\left(\frac{v_{CM}}{R/2}\right)^2 \qquad \left(v_{CM} = \frac{R}{2}\omega\right)$$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{3}{8}m\,v_{CM}^2 \Rightarrow v_{CM} = \sqrt{\frac{14}{3}gR}$$

5. If the body rolls down the inclined plane. The friction should be static and acts in upward direction. Let a be the linear acceleration of centre of mass and α be the angular acceleration of the body.



From force diagram:

for linear motion parallel to the plane

$mg\sin\theta - f = ma$

for rotation around the axis through centre of mass net torque $= I\alpha$

$\Rightarrow \quad fR = mk^2\alpha$

As there is no slipping, the point of contact of the body with plane is instantaneously at rest.

$\Rightarrow \quad v = R\omega$ and $a = R\alpha$

Solve the following three equations for a and f:

$mg\sin\theta - f = ma$

$fR = mk^2\alpha$

$a = R\alpha$

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}} \text{ and } f = \frac{mg\sin\theta}{1 + \frac{R^2}{k^2}}$$

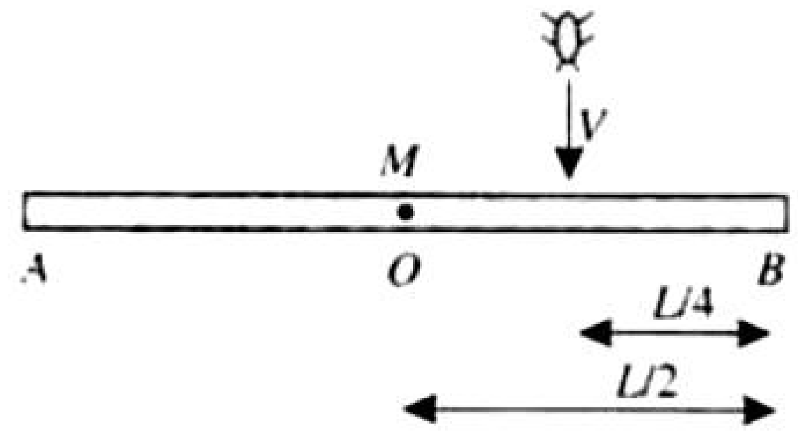
We can also derive the condition for pure rolling (rolling without slipping):

to avoid slipping, $f \le \mu_s N$

$$\frac{g\sin\theta}{1 + \frac{R^2}{k^2}} \le \mu_s\, mg\cos\theta \quad \Rightarrow \quad \mu_s \ge \frac{\tan\theta}{1 + \frac{R^2}{k^2}}$$

This is the condition on so that the body rolls without slipping.

6. Let us take rod and insect as a system. There is not external impulse acting on the system. Hence we can apply conservation of angular momentum just before and just after collision about the point O.



Angular momentum of the system before collision = angular momentum of the system after collision.

$$MV \times \frac{L}{4} = I\omega$$

Where I is the moment of inertia of the system just after collision and ω is the angular velocity just after collision.

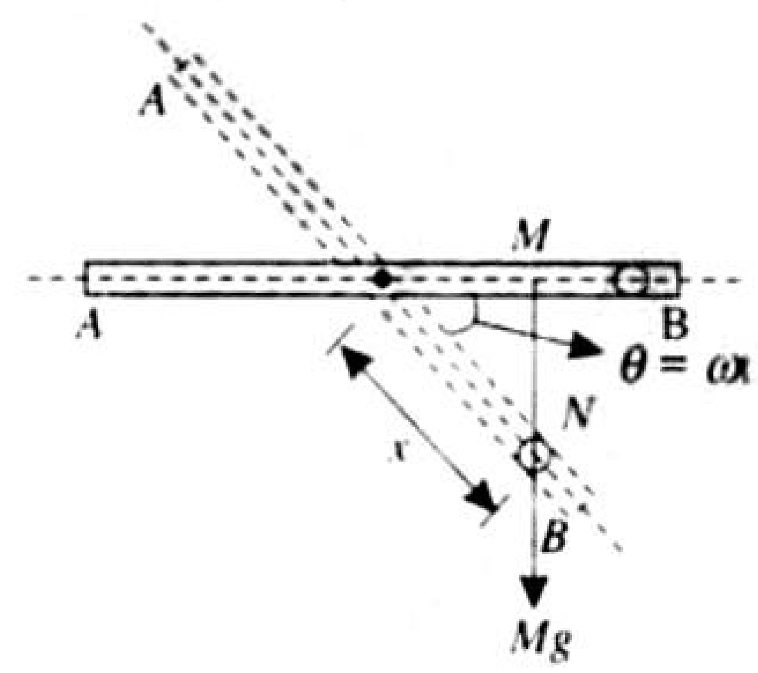
$$\Rightarrow \quad MV \times \frac{L}{4} = \left[M\left(\frac{L}{4}\right)^2 + \frac{1}{12}ML^2\right]\omega$$

$$\Rightarrow \quad MV \times \frac{L}{2} = \frac{ML^2}{4}\left[\frac{1}{4} + \frac{1}{3}\right]\omega = \frac{ML^2}{4}\left[\frac{3+4}{12}\right]\omega$$

$$= \frac{ML^2}{4} \times \frac{7}{12} \times \omega \Rightarrow \omega = \frac{12}{7} \times \frac{V}{L}.$$

Initially the rod was in equilibrium. But after collision there is an extra mass M of the insect which creates a torque in the clockwise direction, which tries to create angular acceleration in the rod. But the same is compensated by the movement of insect towards B due to which moment of inertia of the system increases.

Let at any instant of time t the insect be at a distance x from the center of the rod and the rod has turned through an angle $\theta (= \omega t)$ w.r.t its original position, I.



Torque acting on rod $\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = \omega\frac{dI}{dt}$

$$= \omega\frac{d}{dt}\left[\frac{1}{12}ML^2 + Mx^2\right] = 2M\omega x\frac{dx}{dt} \qquad \text{(i)}$$

This torque is balanced by the torque due to weight of insect τ = Force × Perpendicular distance of force with axis of rotation

$$= Mg \times (OM) = Mg \times x\cos\theta \qquad \text{(ii)}$$

From (i) and (ii)

$$2M\omega x\frac{dx}{dt} = Mg \times x\cos\theta$$

$$\Rightarrow \quad dx = \left(\frac{g}{2\omega}\right)\cos\omega t\,dt$$

On integration taking limits

$$\int_{1/4}^{1/2} dx = \frac{g}{2\omega}\int_0^{\pi/2\omega} \cos \omega t dt; \quad \text{when } x = \frac{L}{4}, \omega t = 0$$

$$[x]_{1/4}^{1/2} = \frac{g}{2\omega^2}[\sin \omega t]_0^{\pi/2\omega} \text{ when } x = \frac{L}{2}, \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{L}{2} - \frac{L}{4} = \frac{g}{2\omega^2}\left[\sin\frac{\pi}{2} - \sin 0\right] \Rightarrow \frac{L}{4} = \frac{g}{2\omega^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2g}{L}}$$

But $\omega = \frac{12}{7}\frac{V}{L} \Rightarrow \frac{12}{7}\frac{V}{L} = \sqrt{\frac{2g}{L}} \Rightarrow V = \frac{7}{12}\sqrt{2gL}$

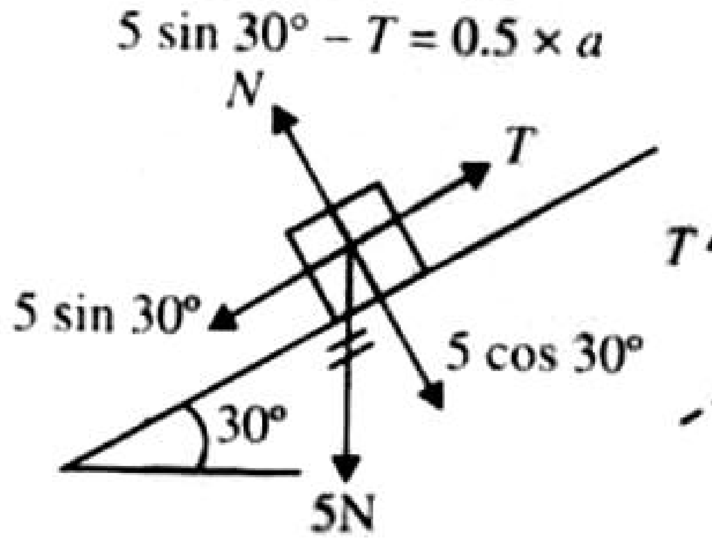
$$\Rightarrow \quad V = \frac{7}{12}\sqrt{2 \times 10 \times 1.8} = 3.5 \text{ ms}^{-1}$$

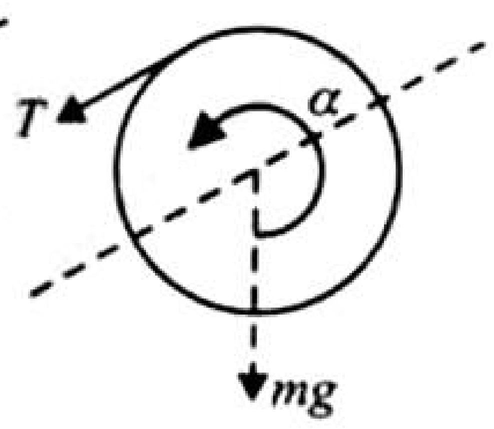
7. a. i. First Method: Force/Torque Method:

Force equation for the block is as follows:

$N - 5 \cos 30° = 0$ (i)

$5 \sin 30° - T = 0.5 \times a$ (ii)





Torque equation for the drum,

$$TR = \left(\frac{1}{2}MR^2\right)\alpha$$

$$\Rightarrow \quad T = \frac{1}{2}MR\alpha = \frac{1}{2} \times 2 \times 0.2\alpha \Rightarrow T = 0.2\alpha \quad \text{(iii)}$$

Here, $a = R\alpha$ (The string does not slip on the drum)

Thus $T = (0.2)\left(\frac{a}{R}\right) = 0.2\left(\frac{a}{0.2}\right) = 1 \times a$ (iv)

From Eqs. (ii) and (iv), we get

$$a = \frac{5}{3} \text{ m/s}, T = \frac{5}{3} \text{ N}$$

ii. Second Method: Using the equation $\vec{\tau} = d\vec{L}/dt$

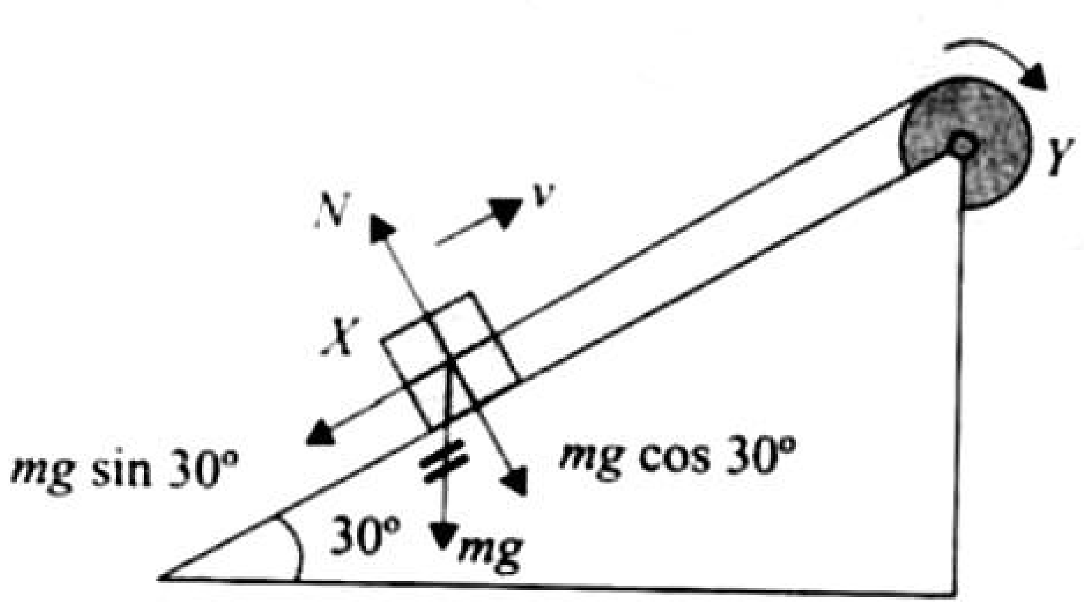
Take both block X and drum Y as a system.

Torque acting on the system about the centre of mass of the drum = $(mg \sin 30°)R$, anticlockwise. Angular momentum of the system about the centre of mass of the drum,

$$L = mvR + \frac{1}{2}MR^2\omega$$

$$= m(R\omega)R + \frac{1}{2}MR^2\omega$$

$$= \left(mR^2 + \frac{1}{2}MR^2\right)\omega, \text{ clockwise}$$



Apply equation $\vec{\tau} = \frac{d\vec{L}}{dt}$

$$-mgR \sin 30° = \frac{d}{dt}\left(mR^2\omega + \frac{1}{2}MR^2\omega\right)$$

$$= \left(mR^2 + \frac{1}{2}MR^2\right)\frac{d\omega}{dt}$$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{-mgR \sin 30°}{\left(mR^2 + \frac{MR^2}{2}\right)}$$

$$\Rightarrow \quad \left(\frac{d\omega}{dt}\right)_{\text{anticlockwise}} = \frac{mgR \sin 30°}{\left(mR^2 + \frac{MR^2}{2}\right)}$$

(ω has been taken in the clockwise sense, $d\omega/dt$ is negative. So, $d\omega/dt$ is in the anticlockwise sense.)

Clearly, $\tau = I\alpha$ $TR = I\left(\frac{d\omega}{dt}\right)_{\text{anticlockwise}}$

$$= \left(\frac{1}{2}MR^2\right)\left(\frac{mgR \sin 30°}{mR^2 + \frac{MR^2}{2}}\right)$$

$$\Rightarrow \quad T = \left(\frac{1}{2}M\right)\left(\frac{mg \sin 30°}{m + \frac{M}{2}}\right) = 5/3 \text{ N}$$

b. i. First Method:

Angular velocity of $Y = 10$ rad/s

Velocity of block $X = R\omega = 0.2 \times 10 = 2$ m/s, along the plane upwards

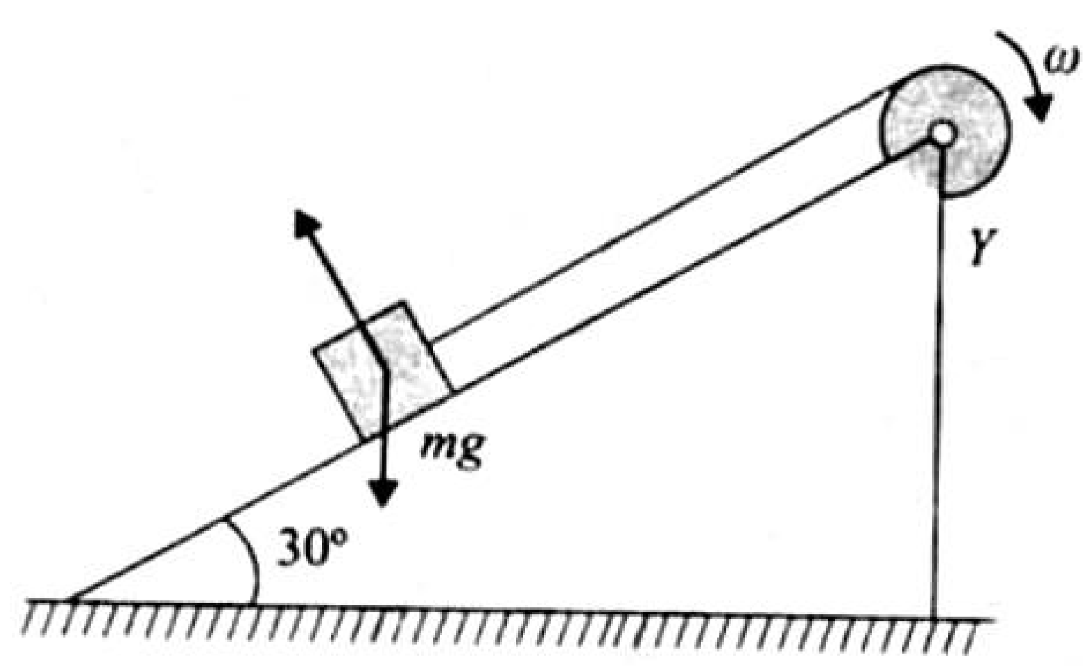
Acceleration of $X = 5/3$ m/s^2 along the plane downwards

Using equation $v^2 = u^2 - 2as$,

$$0^2 = (2) - 2 \times 5/3 \times s \Rightarrow s = \frac{4 \times 3}{2 \times 5} = 1.25 \text{ m}$$

ii. Second Method:

You can also calculate the distance X moving upwards from the energy considerations:



$$W_N + W_{mg} + W_F + W_{Mg} = K_2 - K_1$$

$$0 - (mg \sin \theta)\, s + 0 + 0 = \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$\Rightarrow \quad (mg \sin \theta)\, s = \frac{1}{2}mv^2 + \frac{1}{2I\omega^2}$$

$$\therefore \quad s = \frac{mv^2 + I\omega^2}{2mg \sin \theta}$$

$$= \frac{(0.5)(2)^2 + \frac{1}{2} \times 2 \times (0.2)^2 \times (10)^2}{2 \times 0.5 \times 10 \times \frac{1}{2}} = 1.2 \text{ m}$$

iii. Third Method: By Mechanical Energy Conservation

Here, the only force which does work on the system mg, the algebraic work done by other forces is zero, is conservative in nature; therefore, the mechanical energy of the system will

remain constant. Let the distance moved by the block X along the plane be s.
Gain in gravitational potential energy of $X = mgs \sin \theta$.
Loss in the kinetic energy of the block and the drum is

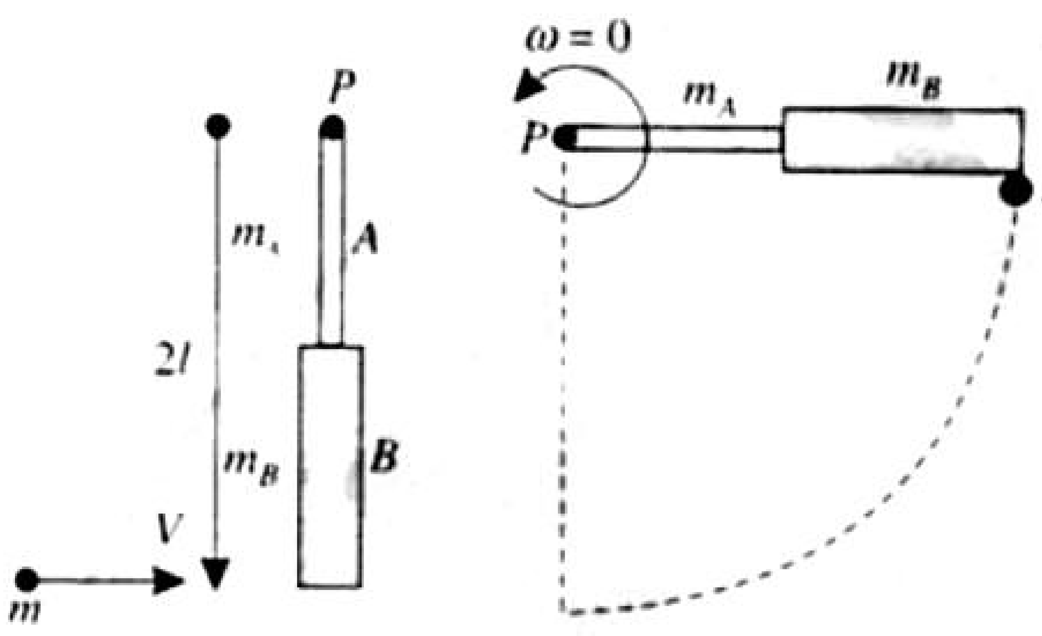
$$\frac{1}{2} mv_0^2 + \frac{1}{2} I\omega_0^2 \; (v_0 = \omega_0 r)$$

Thus, $mgs \sin \theta = \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega_0^2$

$$s = \frac{mv_0^2 + I\omega_0^2}{2\, mg \sin \theta}$$

After substituting the given values, $s = 1.02$ m.

8. Considering rods A and B and object as system. Initially no torque was acting as the system.



Since $\tau = \frac{dL}{dt}$ and $\tau = 0$

The angular momentum just before and just after collision should be conserved.

$\Rightarrow \quad L =$ constant.

Angular momentum just before collision $= (mv) \times 2l$ (i)

Angular momentum after collision $= I\omega$ (ii)

where I is the moment of inertia of the system after collision about P and ω is the angular velocity of the system.

M.I. about P, $I_P =$ M.I. of mass m + M.I. of rod m_A + M.I. of rod m_B

$$I_P = I_1 + I_2 + I_3$$

M.I. of mass m $I_1 = m\, 2l^2$

M.I. of rod m_A $I_2 = m_A\left(\frac{l^2}{12}\right) + m_A\left(\frac{l}{2}\right)^2$

M.I. of rod m_A $I_3 = m_B\left(\frac{l^2}{12}\right) + m_B\left(l + \frac{l}{2}\right)^2$

$$I_P = \left[4ml^2 + m_A\left(\frac{l^2}{12} + \frac{l^2}{4}\right) + m_B\left(\frac{l^2}{12} + \frac{9l^2}{4}\right)\right]$$

$$= \left[4ml^2 + \frac{1}{3}m_A l^2 + \frac{7}{3}m_B l^2\right] = 0.09 \text{ kg-m}^2$$

From (i) and (ii) $\quad I\omega = mv \times 2l$

$$\Rightarrow \quad \omega = \frac{mv \times 2l}{I} = \frac{0.05 \times v \times 2 \times 0.6}{0.09} = 0.67v$$

Applying conservation of mechanical energy just before and just after collision. Loss of K.E. = Gain in P.E.

$$\frac{1}{2} I\omega^2 = mg(2l) + m_A\left(\frac{1}{2}\right)g + m_B g\left(\frac{3l}{2}\right)$$

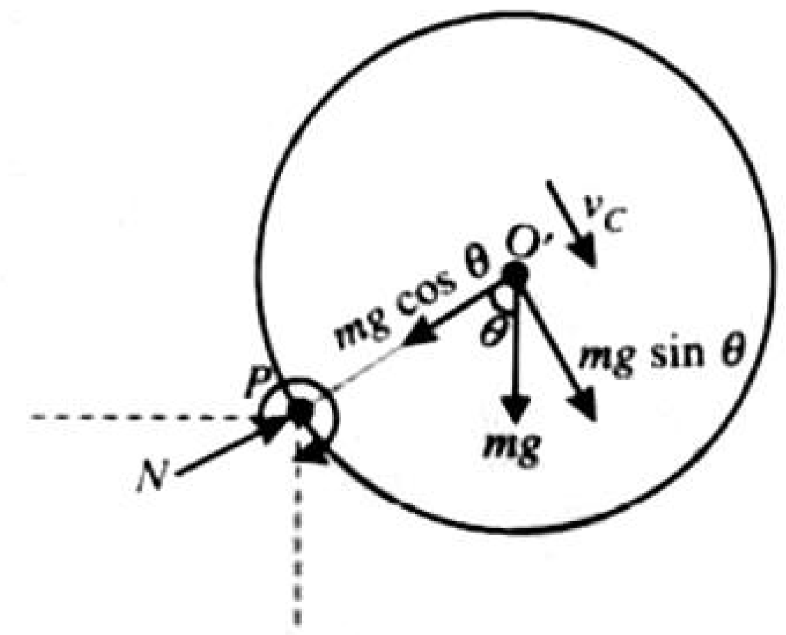
$$\Rightarrow \quad \frac{1}{2} \times 0.09 \times (0.67v)^2$$

$$= \left[0.05 \times 2 + 0.01 \times \frac{1}{2} + 0.02 \times \frac{3}{2}\right] \times 9.8 \times 0.6$$

$\Rightarrow \quad v = 6.3$ m/s

9. Let the initial position of center of mass of the cylinder be O. While rolling down off the edge, let the cylinder be at such a position that its center of mass is at a position O′. As the cylinder is rolling, the c.m. rotates in a circular path. The centripetal force required for the circular motion is given by the equation.

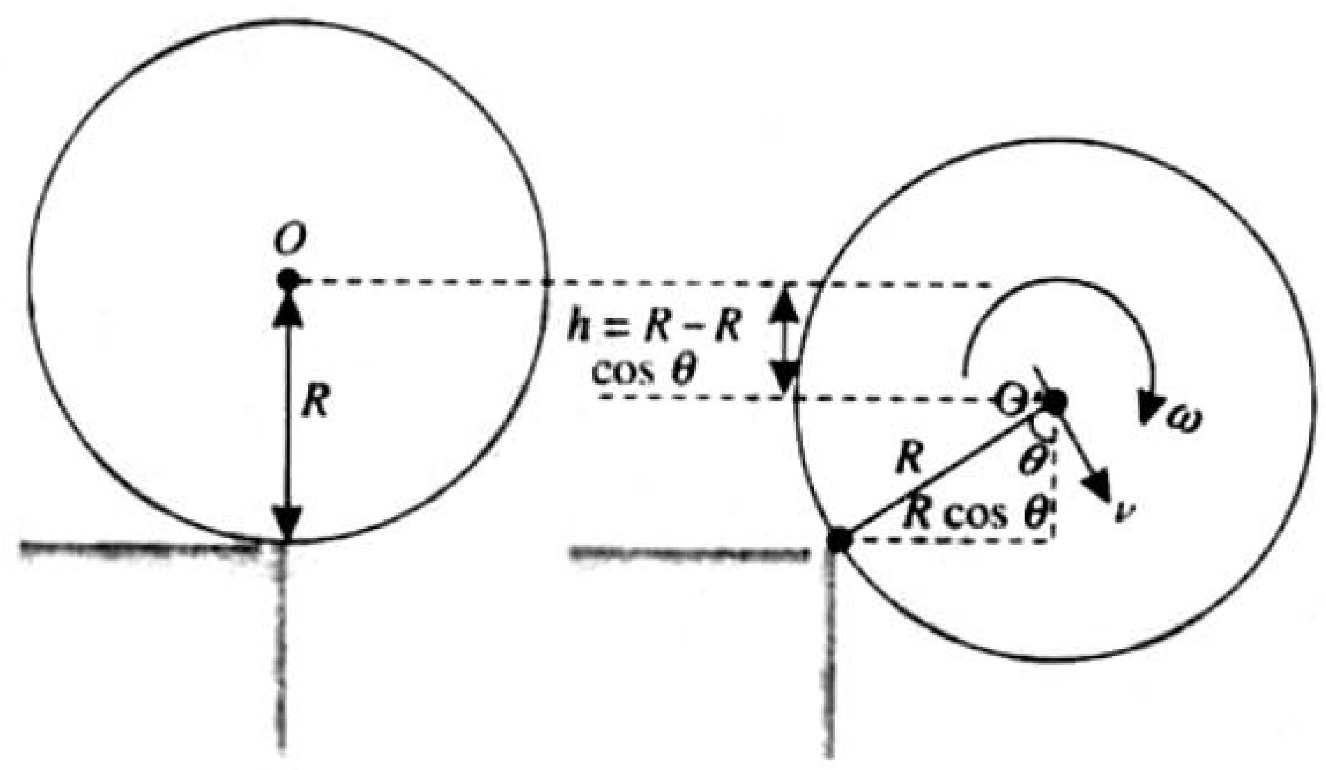
$$mg \cos \theta - N = \frac{mv_C^2}{R}$$



Where N is the normal reaction and m is the mass of cylinder.
If the cylinder leaves the edge $N = 0$

$$mg \cos \theta = \frac{mv_c^2}{R} \Rightarrow \cos \theta = \frac{v_c^2}{Rg} \quad \text{(i)}$$

Applying conservation of mechanical energy from O to O'.
Loss of potential energy of cylinder = Gain in translation kinetic energy + Gain in rotational kinetic energy.



$$mgh = \frac{1}{2} mv_C^2 + \frac{1}{2} I\omega^2 \quad \text{(ii)}$$

where I is the moment of inertia of the cylinder about its axis of rotation passing through O', ω is the angular speed, v_c is the velocity of its center of mass.

For rolling $\quad v_c = \omega R \Rightarrow \omega = \frac{v_C}{R}$ (iii)

From (ii) and (iii), we get

$$mgh = \frac{1}{2} mv_c^2 + \frac{1}{2} \times \left(\frac{1}{2} mR^2\right) \times \frac{v_c^2}{R^2}$$

$$\Rightarrow \quad gh = \frac{1}{2} v_C^2 + \frac{1}{5} v_C^2 = \frac{3}{4} v_C^2 \Rightarrow v_C^2 = \frac{4gh}{3}$$

$$\therefore \quad v_C^2 = \frac{4g}{3} R(1 - \cos \theta) \quad \text{(v)}$$

From (i) and (v) we get

$$\cos\theta = \frac{4gr}{3Rg}(1-\cos\theta) \Rightarrow 3\cos\theta = 4 - 4\cos\theta$$

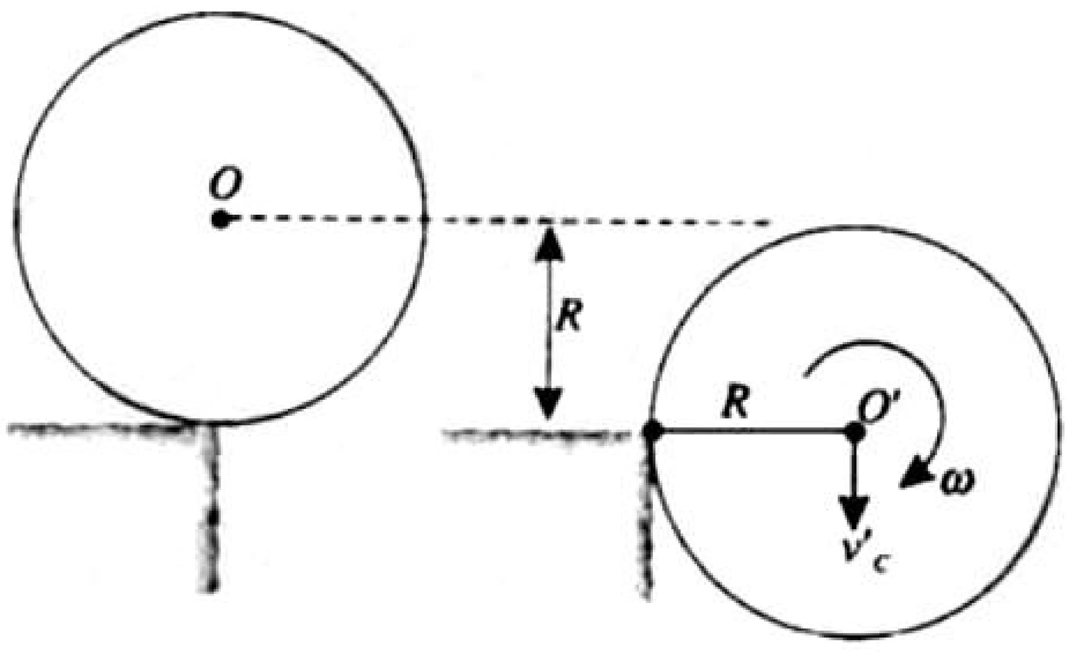
$$\Rightarrow \quad \cos\theta = \frac{4}{7}$$

(b) Speed of center of mass of cylinder before leaving contact with edge

From (v) $v_C^2 = \frac{4gR}{3}\left[1-\frac{4}{7}\right] = \frac{4gR}{7} \Rightarrow v_C = \sqrt{\frac{4gR}{7}}$

(c) Before the cylinder's center of mass reaches the horizontal line of the edge, it leaves contact with the edge as $\theta = \cos^{-1}\left(\frac{4}{7}\right)$

Therefore the rotational KE, which the cylinder gains at the time of leaving contact with the edge remains the same in its further motion. Thereafter, the cylinder gains translation K.E. Again applying energy conservation from O to the point where center of mass is in horizontal line with edge



$$mgR = \frac{1}{2}I\omega^2 + \frac{1}{2}m(v_C')^2$$

$$mgR = \frac{1}{2}\times\frac{1}{2}mR^2\times\left(\sqrt{\frac{4g}{7R}}\right)^2 + \frac{1}{2}m(v_C')^2$$

$$\because \quad \omega = \frac{v_C'}{R} = \sqrt{\frac{\frac{4gR}{7}}{R}}$$

$$\Rightarrow \quad mgR - \frac{mgR}{7} = \text{Translational K.E.} = \frac{6mgR}{7}$$

Also rotational KE $= \frac{1}{2}I\omega^2 = \frac{mgR}{7}$

$$\therefore \quad \frac{\text{Translational KE}}{\text{Rotational KE}} = 6$$

10. FBD of the disc.

When the disc is projected it starts sliding and hence there is a relative motion between the points of contact. Therefore frictional force acts on the disc in the direction opposite to the motion.

a. Now for translational motion

$$\vec{a}_{cm} = \frac{\vec{f}}{m}$$

$f = \mu N$ (as it slides) $= \mu mg$

$\Rightarrow \quad a_{cm} = -\mu g$, negative sign indicates that a_{cm} is opposite to v_{cm}

$$\Rightarrow \quad v_{cm(t)} = v_0 - \mu g t_0$$

$$\Rightarrow \quad t_0 = \frac{(v_0 - v)}{\mu g}, \text{ where } v_{cm(t_0)} = v \qquad \text{(i)}$$

For rotational motion about centre

$$\tau_f + \tau_{mg} = I_{cm}\alpha$$

$$\Rightarrow \quad \mu mgr = \frac{mr^2}{2}\alpha$$

$$\Rightarrow \quad \alpha = \frac{2\mu g}{r} \qquad \text{(ii)}$$

Therefore $\omega_{(t_0)} = 0 + \frac{2\mu g}{r}t_0$, using $\omega_t = \omega_0 + \alpha t$

$$\Rightarrow \quad \omega = \frac{2(v_0 - v)}{r} \qquad \text{(iii)}$$

using (i), $v_{cm} = \omega r \Rightarrow v = 2(v_0 - v)$ using (iii)

$$\Rightarrow \quad v = \frac{2}{3}v_0$$

b. Putting the value of v in Eq. (i), we get

$$t_0 = \frac{v_0}{3\mu g}$$

c. Work done by the frictional force is equal to change in KE

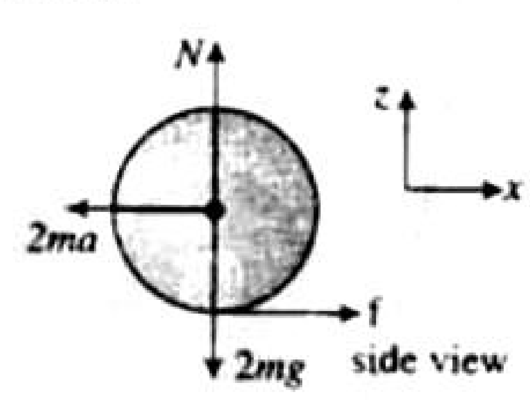
$$W_{\text{friction}} = \frac{1}{2}m(v_0 - \mu gt)^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{2\mu gt}{r}\right)^2 - \frac{1}{2}mv_0^2$$

d. For time $t > t_0$, work done by the friction is zero. Therefore, for longer time total work done is the same as that in part (c)

$$\Rightarrow \quad W = -m v_0^2/6$$

11. a. FBD of the object with respect to truck.

In the reference frame of truck it experiences a pseudoforce



$$\vec{F} = -2ma\hat{i}$$

where a = acceleration of the truck.

Pseudoforce does not provide torque about the centre of the disc. Because of this force, object has tendency to slide along –ve x-axis. Hence frictional force will act along +ve x-axis.

For translational motion

$$2ma - f = 2ma' \qquad \text{(i)}$$

Here, a' = acceleration of the centre of mass of the object.

For rotational motion, $fR = I\alpha$

For no slipping, $\alpha = \frac{a'}{R}$

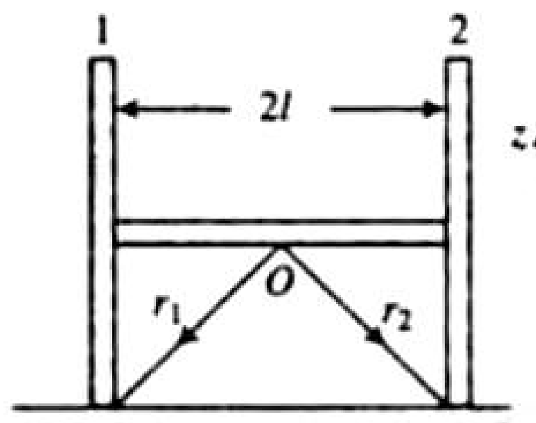
Hence, $f.R = 2\left(\frac{mR^2}{2}\right)\left(\frac{a'}{R}\right)$

$$\Rightarrow \quad a' = \frac{f}{m} \qquad \text{(ii)}$$

From Eqs. (i) and (ii) we get $f = \frac{2}{3}ma\hat{i}$

Therefore, force of friction on each disc is

$$\frac{f}{2} = \frac{ma}{3}\hat{i} = 6\hat{i}\ \text{N}$$



b. $\vec{f}_1 = \frac{ma}{3}\hat{i}$

$$\vec{r}_1 = -l\hat{j} - R\hat{k} \Rightarrow \tau_{(f_1)} = \vec{r}_1 \times \vec{f}_1$$

$$= -(l\hat{j} + R\hat{k}) \times \frac{ma}{3}\hat{i}$$

$$= -\frac{maR}{3}\hat{j} + \frac{mal}{3}\hat{k}$$

$$= -6 \times 0.1\hat{j} + 6 \times 0.1\,\hat{k}$$

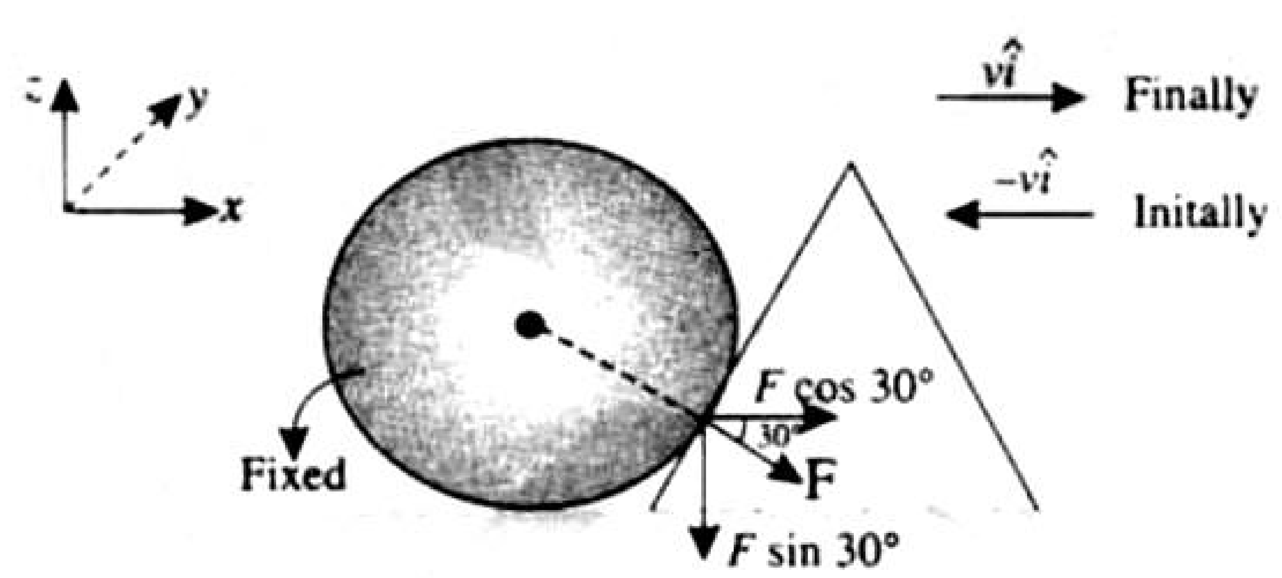
$$= -0.6\hat{j} + 0.6\hat{k}$$

$$\tau_{(f_2)} = -0.6\hat{j} - 0.6\hat{k}$$

c. Maximum value of frictional force is $2\mu mg$

$$\Rightarrow \quad \frac{2}{3}ma \le 2\mu mg \Rightarrow \mu \ge \frac{a}{3g}$$

12. a. (i) The collision between sphere and wedge is elastic. As the sphere is fixed, hence the wedge will return with velocity $v\hat{i}$.



Now, linear impulse in x-direction
= change in momentum in x-direction.

$\therefore \quad (F\cos 30°)\Delta t = mv - (-mv) = 2mv$

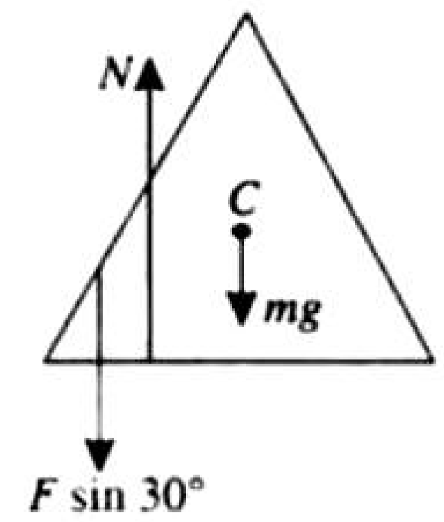
$$\therefore \quad F = \frac{2mv}{\Delta t \cos 30°} = \frac{4mv}{\sqrt{3}\Delta t}$$

$$F = \frac{4mv}{\sqrt{3}\Delta t}$$

$$\therefore \quad \vec{F} = (F\cos 30°)\hat{i} - (F\sin 30°)\hat{k}$$

$$\vec{F} = \left(\frac{2mv}{\Delta t}\right)\hat{i} - \left(\frac{2mv}{\sqrt{3}\Delta t}\right)\hat{k}$$

(ii) Now consider the equilibrium of wedge in vertical direction.



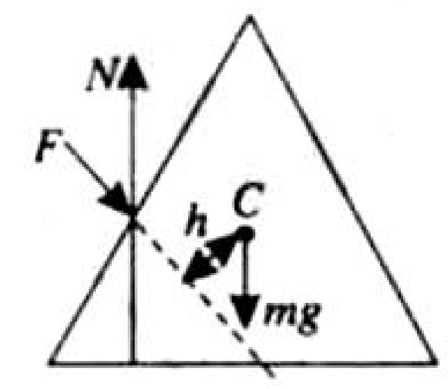
During collision.

$$N = mg + F\sin 30°$$

$$N = mg + \frac{2mv}{\sqrt{3}\Delta t}$$

or in vector form $\vec{N} = \left(mg + \frac{2mv}{\sqrt{3}\Delta t}\right)\hat{k}$

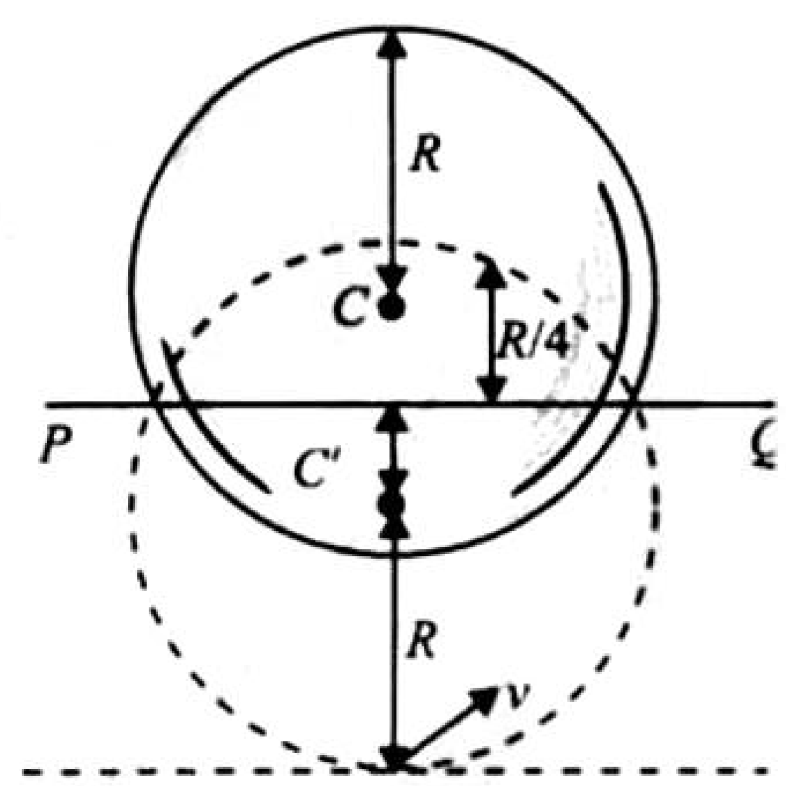
b. For rotational equilibrium of wedge about its (CM), anticlockwise torque of F = clockwise torque due to N



$\therefore$ Magnitude of torque of N about center of mass = magnitude of torque of F about center of mass = $F \cdot h$

$$|\vec{\tau}_N| = \left(\frac{4\,mv}{\sqrt{3}\Delta t}\right)h$$

13. During the fall, the disc–mass system gains rotational kinetic energy. This is at the expense of potential energy.



Reference
level of PE

Applying conservation of mechanical energy, we get
Initial total energy = Final total energy

$$mg\left(2R + \frac{2R}{4}\right) + mg\left(R + \frac{2R}{4}\right) = mgR + \frac{1}{2}I\omega^2$$

where I = MI of the disc–mass system about PQ

$$= mg + \frac{10R}{4} + mg\,\frac{6R}{4} = mgR + \frac{1}{2}\omega^2$$

$$\Rightarrow \quad 3mgR = \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6mgR}{I}} \qquad \text{(i)}$$

$$(I)_{PQ} = (I_{\text{disc}})_{PQ} + (I_{\text{mass}})_{PQ}$$

$$= \left[\frac{mR^2}{4} + M\left(\frac{R}{4}\right)^2\right] + m\left(\frac{5R}{4}\right)^2$$

[Therefore, MI of disc about the diameter = (1/4) MR^2]

$$= \frac{mR^2[4+1+25]}{16} = \frac{15mR^2}{8} \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\omega = \sqrt{\frac{6mgR \times 8}{15mR^2}} = \sqrt{\frac{16g}{5R}}$

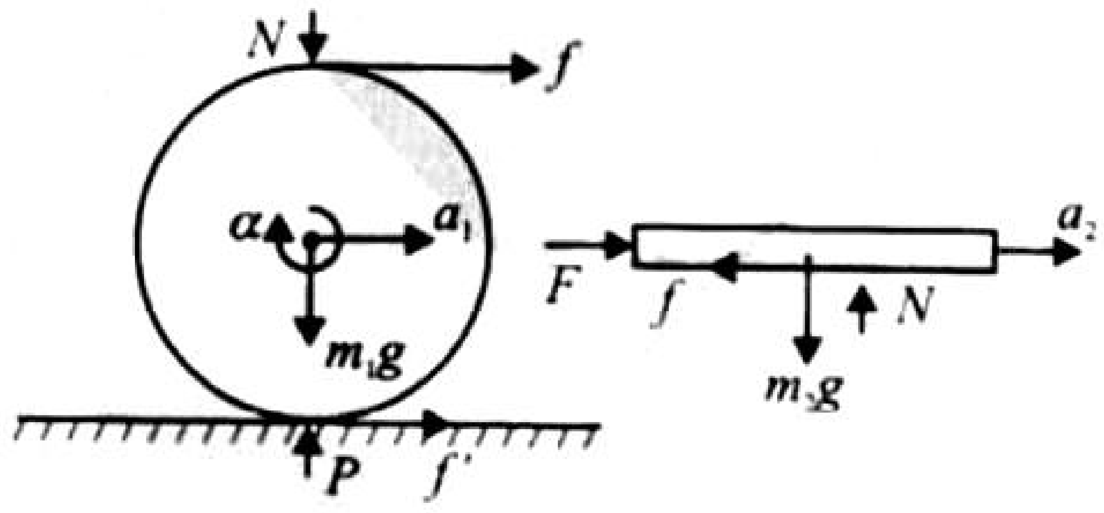
Let v be the velocity of mass m at the lowest point of rotation

$$v = \omega\left(R + \frac{R}{4}\right)$$

$$= \sqrt{\frac{16g}{5R}} \times \frac{5R}{4} = \sqrt{5gR}$$

14. ***Analysis of the direction of friction force (f)***

The man applies a force F in the horizontal direction on the plank as shown in the figure. Therefore, the point of contact of the plank with the cylinder will try to move towards the right. Therefore, the friction force f will act towards the left on the plank.



To each and every action, there is an equal and opposite reaction. Therefore, a frictional force f will act on the top of the cylinder towards the right. Direction of f:

For finding the direction of f' first assume f' is not acting.

A force f is acting on the cylinder. This force is trying to move the point of contact towards right by an acceleration $a_{CM} = f/m_1$ acting towards the right.

At the same time, the force f is trying to rotate the cylinder about its centre of mass in the clockwise direction.

$$f \times R = I \times \alpha \Rightarrow \alpha = \frac{f \times R}{I} = \frac{f \times R}{\frac{1}{2}m_1R^2} = \frac{2f}{m_1R}$$

Therefore, acceleration of the point of contact,

$$a_p = a_{CM} - \alpha R = \frac{f}{m_1} - \frac{2f}{m_1R} \times R = \frac{f}{m_1}$$

i.e., towards the left.

Therefore, the point of contact of the cylinder with the ground moves towards the left. Therefore, friction force acts towards the right on the cylinder.

Here $k^2 < Rx$ as $k^2 = \frac{R^2}{2}$

and $Rx = R \cdot R = R^2$

Hence, friction should act in forward direction.

Applying Newton's law on the plank, we get

$F - f = m_2a_2$ (i)

Also $a_2 = 2a_1$ (ii)

because a_2 is the acceleration of the topmost point of the cylinder and there is no slipping. Applying Newton's law on the cylinder

$m_1a_1 = f + f'$ (iii)

The torque equation for the cylinder is

$$f \times R - f' \times R = I\alpha = \frac{1}{2}m_1R^2 \times \left(\frac{a}{R}\right)$$

$\because \quad I = m_1R^2$ and $R\alpha = a_1$

$\therefore \quad (f - f')R = \frac{1}{2}m_1Ra_1$

$\therefore \quad f + f' = \frac{1}{2}m_1Ra_1$ (iv)

Solving Eqs. (iii) and (iv), we get $f = \frac{3}{4}m_1a_1$ (v)

and $f' = \frac{1}{4}m_1a_1$ (vi)

From Eqs. (i) and (iii), $F - f = 2m_2a_1$

$\Rightarrow \quad F = \frac{3}{4}m_1a_1 = 2m_2a_1$

$\therefore \quad a_1 = \frac{4F}{3m_1 + 8m_2}$

$\therefore \quad a_2 = \frac{8F}{3m_1 + 8m_2}$

From Eqs. (v) and (vi), we get

$$f = \frac{3}{4}m_1 \times \frac{4F}{3m_1 + 8m_2} = \frac{3Fm_1}{3m_1 + 8m_2}$$

and $f' = \frac{1}{4}m_1 \times a_1 = \frac{3Fm_1}{3m_1 + 8m_2}$

15. a. If we take rod and particle as a system, the impulse between them will be internal. We can conserve the linear momentum of rod + particle's system. Let V_c and ω be the velocity and angular velocity of the CM just after collision.

Using conservation of linear momentum, we have

$mv_0 = 0 + Mv_c$ (i)

Here in this case we can conserve angular momentum about any point on the rod.

And by conservation of angular momentum about the entire of mass of the rod, we get

$$mv_0\frac{L}{2} = I\omega \text{ or } mv_0\frac{L}{2} = \left(\frac{ML^2}{12}\right)\omega \quad \text{(ii)}$$

From Eqs. (i) and (ii), $v_c = \frac{mv_0}{M}$ and $\omega = \frac{6mv_0}{ML}$

We can apply conservation of angular momentum about point of collision.

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

$$O = I_0\omega(-\hat{k}) + Mv_0\frac{l}{2}(\hat{k})$$

$$\frac{ML^2}{12}\omega = \frac{Mv_cL}{2}$$

which gives $\omega L = 6v_c$ (iii)

Solving Eqs. (i) and (iii), we get the same results.

Since collision is completely elastic, therefore KE before collision is equal to KE after collision

or $\quad \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$

or $\quad \frac{1}{2}mv_0^2 = \frac{1}{2}M\left(\frac{mv_0}{M}\right)^2 + \frac{1}{2}\frac{ML^2}{12}\left(\frac{6mv_0}{ML}\right)^2$

which gives $\frac{m}{M} = \frac{1}{4}$ and $v_c = \frac{v_0}{4}$

Alternative method: We can apply coefficient of restitution equation at the position of point of collision, we have

$$e = \left(\frac{v_2 - v_1}{u_1 - u_2}\right)$$

Here u_1, u_2, v_1 and v_2 are the velocities of particles and point on the rod at collision point.

$u_1 = v_0$, $u_2 = 0$

$v_1 = 0$ and $v_2 = v_c + \frac{\omega l}{2}$

As collision in elastic $e = 1$.

Substituting the values in restitution equation, we get

$$1 = \frac{\left(v_c + \frac{\omega L}{2}\right) - 0}{v_0 - 0}$$

$$\Rightarrow v_0 = \left(\frac{mv_0}{M}\right) + \left(\frac{6mv_0}{ML}\right)\frac{L}{2}$$

which gives $\frac{m}{M} = \frac{1}{4}$

b. Velocity of point P immediately after collision is zero, let it be at a distance y from CM

$\therefore \quad v_c - \omega y = 0$ or $\frac{mv_0}{M} - \left(\frac{6mv_0}{ML}\right) y = 0$

which gives $y = \frac{L}{6}$

$\therefore \quad AP = \frac{L}{2} + \frac{L}{6} = \frac{2L}{3}$

c. Angle rotated by the rod at time $\frac{\pi L}{3v_0}$

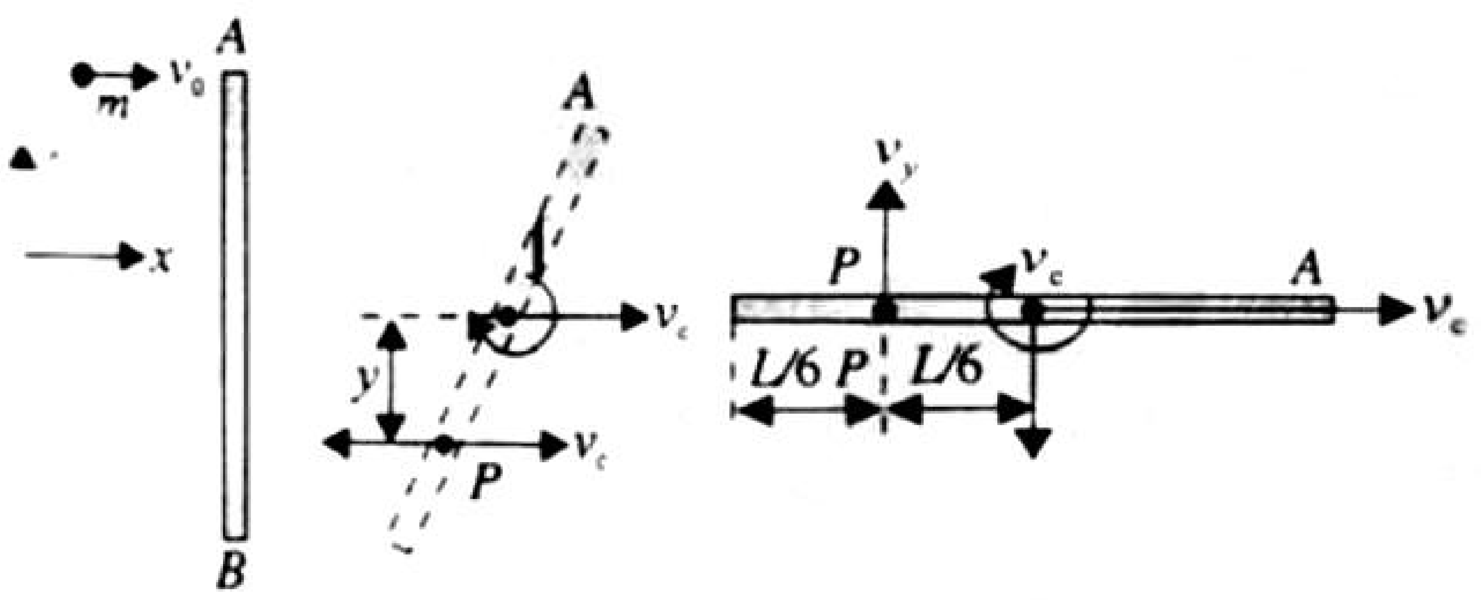
$$\theta = \omega t = \left(\frac{6mv_0}{ML}\right) \times \frac{\pi L}{3v_0} = \frac{\pi}{2}$$

The rod turns through $\pi/2$ in this interval of time. The velocity of point P in y-direction will be

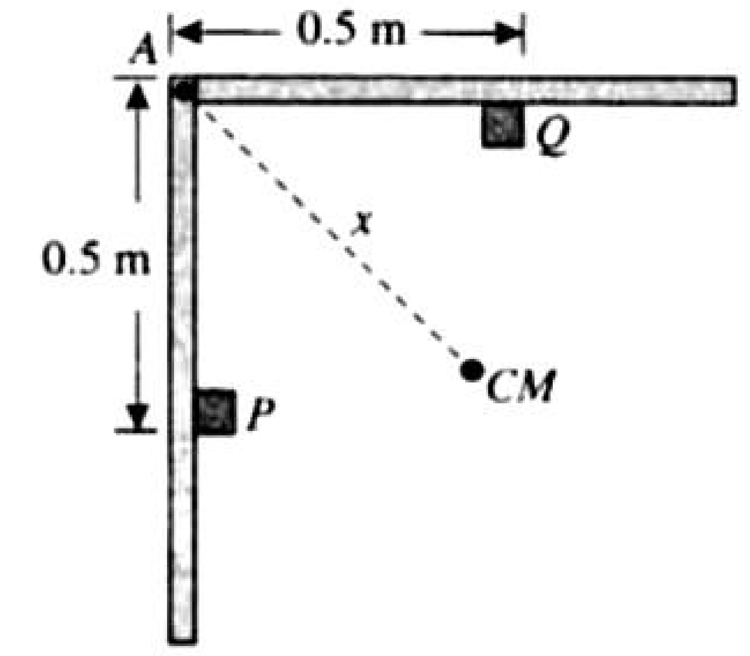
$$v_y = \omega y = \left(\frac{6mv_0}{M}\right) \times \frac{L}{6} = \frac{v_0}{4}$$

The resultant velocity of point P,

$$v = \sqrt{v_c^2 + v_y^2} = \sqrt{\left(\frac{v_0}{4}\right)^2 + \left(\frac{v_0}{4}\right)^2} \Rightarrow \frac{v_0}{2\sqrt{2}}$$



16. Let x be the distance of center of mass from AB.



Moment of inertia of the laminar sheet about an axis passing through center of mass and parallel to AB = 1.2 kg m^2

$I_{AB} = I_{cm} + Mx^2$ (i)

Impulse $= M v_2 - (-v_1) = M(v_2 + v_1)$

$6 = M\ \omega_2 x + \omega_1 x = Mx(\omega_2 + \omega_1)$ (ii)

where ω_1 and ω_2 are the angular velocities of the sheet before and after the collision with obstacle.

Impulsive torque $= -6 \times 0.5 = L_f - L_i$

$L_i = L_f + 3$

$(L_i - L_f) = 3$

$I_{AB}\ \omega_1 - (-\omega_2) = 3 \Rightarrow I_{AB}(\omega_1 + \omega_2) = 3$ (iii)

From equations (ii) and (iii), $I_{AB} = \frac{Mx}{2}$

$$I_{AB} = \frac{Mx}{2} = 1.2 + Mx^2$$

$$30x^2 - 15x + 1.2 = 0$$

$$x = \frac{15 \pm 9}{60} = 0.4 \text{ or } 0.1 \text{ m}$$

a. Substituting $x = 0.4$ m in equation (ii)

$6 = 30(0.4)(\omega_2 + 1)$

$\omega_2 = -0.5$ (this is not possible)

The laminar sheet turns back after collision

$\therefore \quad x = 0.1$ m

The center of mass is at a distance 0.1 m from AB.

b. Substituting $x = 0.1$m in equation (ii)

$6 = 30\ (0.1)\ (\omega_1 + 1)$

Angular velocity of laminar sheet after collision = 1 rad/s

This is possible if collision is elastic.

c. Since there is no change in kinetic energy, it continues to move, i.e., it makes infinite number of impacts on the obstacles. The laminar sheet will never come to rest.

17.

a. The centre of mass of the system is at the centroid of a triangular assembly. The CM moves along a circular path with constant angular velocity. Therefore, there must be a horizontal centripetal force directed towards the axis at the hinge.

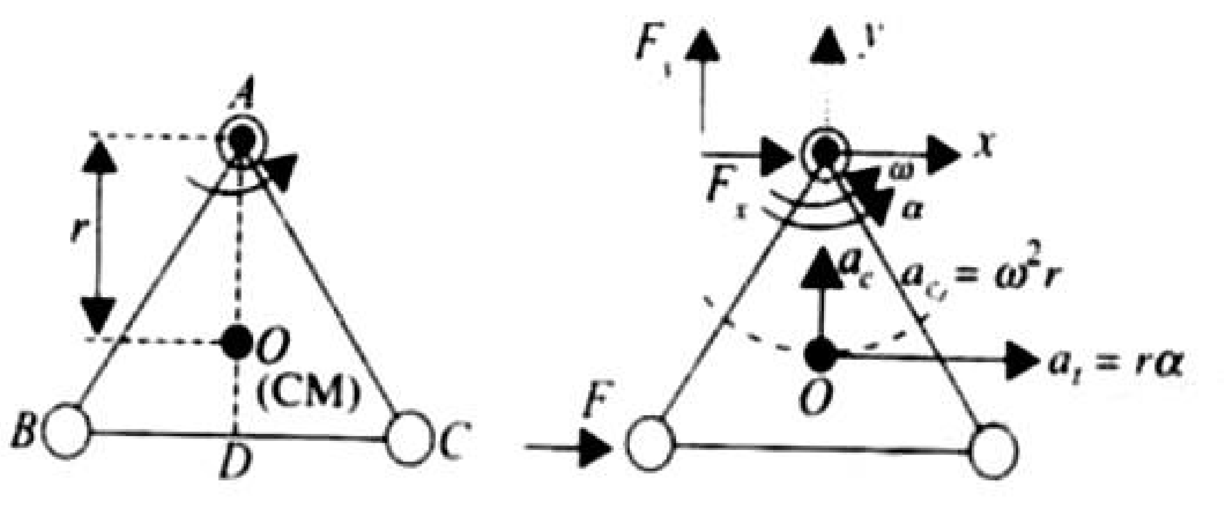
From figure (a), we find

$$AD = l\sin 60° = \frac{l\sqrt{3}}{2}$$

$$AO = \frac{2}{3}AD = \frac{l^2}{\sqrt{3}} = r$$

The centripetal acceleration $a_c = \omega^2 r = \omega^2\left(\frac{l}{\sqrt{3}}\right)$

Tangential acceleration $a_t = \alpha r = \alpha\left(\frac{l}{\sqrt{3}}\right)$



(a) (b)

b. Let F_x and F_y be the forces applied by the hinges along x-axis and y-axis, respectively. The system is in non-centroidal rotation. The three equations of motion are

$$\Sigma F_x = F_x + F = (3m)a_t = 3m\left(\frac{l}{\sqrt{3}}\alpha\right) \quad \text{(i)}$$

$$\Sigma F_y = F_y = 3m\left(\frac{l}{\sqrt{3}}\right)\omega^2 \quad \text{(ii)}$$

$$\Sigma\tau = F \times \left(\frac{\sqrt{3}}{2}l\right) = 2ml^2\alpha \quad \text{(iii)}$$

From Eq. (iii), $\alpha = \frac{\sqrt{3}F}{4ml}$

From Eq. (i), $F_x + F = 3m\left(\frac{l}{\sqrt{3}}\alpha\right) \times \frac{\sqrt{3}F}{4ml} = \frac{3F}{4}$

$\Rightarrow \quad F_x = \frac{F}{4}$

From Eq. (ii), $F_y = \sqrt{3}\, ml\omega^2$

18. We know that $\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow \vec{\tau} \times dt = d\vec{L}$

when angular impulse $(\vec{\tau} \times d\vec{t})$ is zero, the angular momentum is constant. In this case for the wooden log-bullet system, the angular impulse about O is constant.
Therefore,
[angular momentum of the system]$_{\text{initial}}$ = [angular momentum of the system.]$_{\text{final}}$

$\Rightarrow \quad m\upsilon \times L = I_0 \times \omega \quad$ (i)

where I_0 is the moment of inertia of the wooden log-bullet system after collision about O

$$I_0 = I_{\text{wooden log}} + I_{\text{bullet}} = \frac{1}{3}ML^2 + ML^2 \quad \text{(ii)}$$

From (i) and (ii) $\omega = \frac{m\upsilon + L}{\left[\frac{1}{3}ML^2 + ML^2\right]}$

$$\Rightarrow \quad \omega = \frac{m\upsilon}{\left[\frac{ML}{3} + mL\right]} \quad \Rightarrow \quad \omega = \frac{3m\upsilon}{(M + 3m)L}$$

19. Applying $F_{\text{net}} = ma$ in X-direction

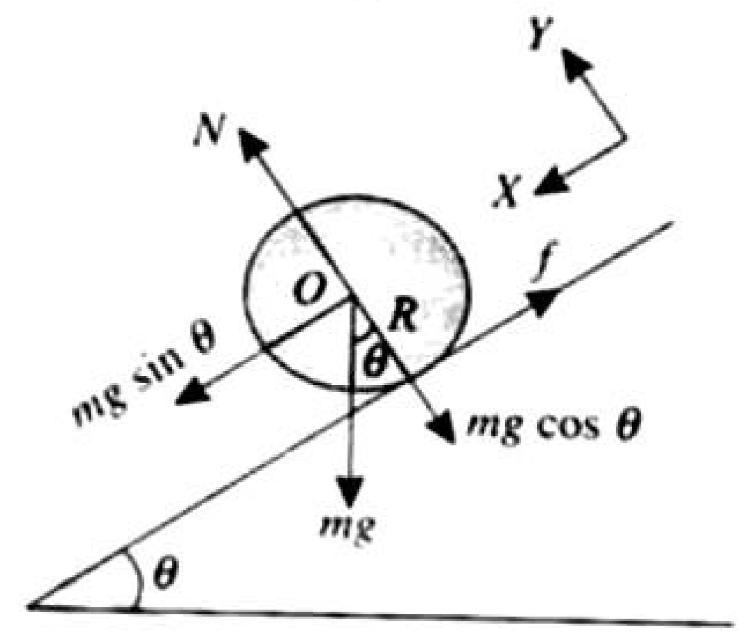
$mg \sin\theta - f = ma \quad$ (i)

The torque about O will be

$\tau = f \times R = I\alpha \quad$ (ii)

As the case is of rolling therefore

$$a = \alpha R \quad \Rightarrow \quad \alpha = \frac{a}{R} \quad \text{(iii)}$$



From (ii) and (iii) $f = \frac{Ia}{R^2}$

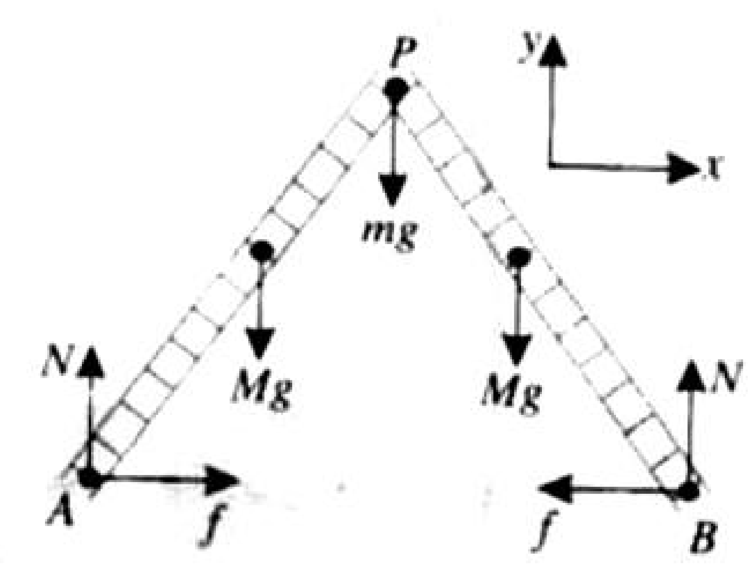
Substituting this value in (i) we get

$$mg\sin\theta - \frac{Ia}{R^2} = ma$$

$$\Rightarrow \quad a = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{mg\sin\theta}{m + \frac{1}{2}\frac{mR^2}{R^2}} = \frac{2}{3}g\sin\theta$$

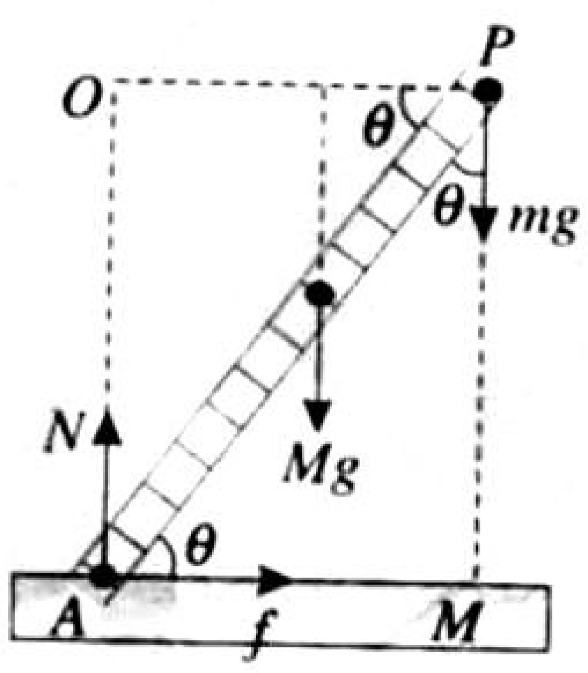
$$\left[\because I = \frac{1}{2}mR^2 \text{ for solid cylinder}\right]$$

20. The various forces acting on the ladders are shown in the figure. Since the system is in equilibrium, therefore $\Sigma F_y = 0$
$\Rightarrow Mg + mg + Mg = N + N$



$$\Rightarrow \quad N = \frac{(2M + m)g}{2} \quad (1)$$

Considering the rotational equilibrium of one ladder as shown in the figure. Calculating torques about P



$$Mg \times P + f \times PM = N \times OP$$

$$Mg \times \frac{L}{2}\cos\theta + f \times L\sin\theta = NL\cos\theta$$

$$\Rightarrow \quad f = \frac{NL\cos\theta - \frac{MgL}{2}\cos\theta}{L\sin\theta} = N\cot\theta - \frac{Mg}{2}\cot\theta$$

$$\Rightarrow \quad f = \left[\left(\frac{2N+m}{2}\right)g - \frac{Mg}{2}\right]\cot\theta$$

$$\Rightarrow \quad f = \left[(M+m)\frac{g}{2}\right]\cot\theta$$

21. Since the plate is held horizontal, therefore net torque acting on the plate is zero.

$$Mg \times \frac{b}{2} = F \times \frac{3b}{4} \qquad \text{(i)}$$

$$F = n\frac{dp}{dt} \times (Area) = n \times (2mv) \times a \times \frac{b}{2} \qquad \text{(ii)}$$

From (i) and (ii)

$$Mg \times \frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$$

$$3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$$

$$\Rightarrow \quad v = 10 \text{ m/s}$$